MULTIVIBRATOR STEP-DOWN FRACTIONAL RATIOS

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Transmission
Research

Frequency step-downs, where the lower frequency is accurately related to some higher frequency, are often needed in communication circuits. Such step-downs are required, for example, in testing filter characteristics, as already described in the RECORD.* Heretofore, moderately complex vacuum tube demodulating circuits have been employed, but for many purposes the multivibrator step-down offers considerable advantage. It gives a much simpler circuit employing fewer apparatus units, and thus is less expensive to build and occupies much less space. These advantages would have led to their wide employment, except that prior to the war, the step-down ratios that could be obtained were limited to integral numbers, and for larger ratios the number could not be a prime. This restriction was removed in the course of war developments by applying feedback to a multivibrator chain. Besides making prime ratios readily obtainable, it also permits the utilization of non-integral rational numbers such as $14\frac{95}{121}$.

A multivibrator circuit capable of acting as a step-down is shown in Figure 1. With both tubes passing current, the circuit is stable. Since the tubes are in a saturated condition, the voltages at points 1 and 3 will be zero, those at points 2 and 4 slightly positive, and capacitors C1 and C2 will have small charges. If anything is done that momentarily interrupts the flow of current in one of the tubes, however, the circuit at once starts to oscillate: first one tube passes current and then the other, and the beginning of current flow in one tube blocks the flow of current in the other.

Assume, for example, that the voltage at point 1 has been momentarily made highly negative, and that as a result tube *a* blocks, while tube *b* continues to pass current. There will be a negative charge on C1 because of the high negative voltage recently

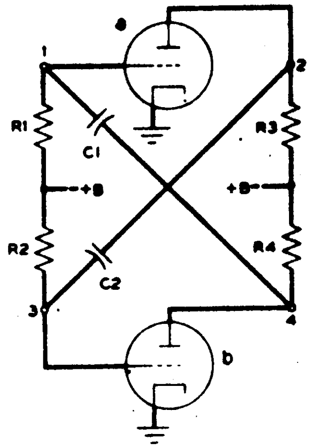

Fig. 1—Schematic of a multivibrator circuit capable of acting as a step-down

applied, but current from positive battery through R1 flowing into the capacitor will slowly raise the voltage at point 1. When the cut-off voltage is reached, tube *a* at once starts to pass current. While *a* was blocked, the voltage at point 2 had risen to full positive battery potential, and capacitor C2 had been fully charged. When *a* starts to pass current, however, the potential at point 2 drops suddenly to nearly zero, and the charge on C2 released as a result, passing through R2, momentarily decreases the voltage at point 3 to a high negative value, and tube *b* blocks as a result. Tube *b* then starts a cycle like that described for *a*, and when *b* starts to pass current, *a* will block.

The frequency of oscillation depends on the duration of the blocked periods of the two tubes, since the conducting period is

*RECORD, *March*, 1935, *page* 263.

stable and tends to continue indefinitely. The duration of the blocked periods of tube a is controlled by R1 and C1, and that for tube b by R2 and C2. With frequency depending on the values of resistors and capacitors, variation is likely, but by associating the output of an oscillator with points 1 and 3, the frequency of the multivibrator can be made as constant as that of the oscillator, but lower by some integral factor.

Suppose, for example, that the voltage of the oscillator, reduced to a small fixed value, were superimposed on the voltages at points 1 and 3. Instead of rising along a smooth curve, the voltages at these points would become as shown by the solid curve of Figure 2. Without the superimposed oscillator voltage, the tube would have started to pass current at t_0, but with it, it starts to pass at t_1—just four cycles of the oscillator frequency after the tube had blocked. Assuming similar constants and arrangements for the other tube, the frequency of the multivibrator would be one-eighth that of the oscillator, since each half cycle is four times as long as one cycle of the oscillator circuit itself.

Greater step-down may be secured by connecting several multivibrator circuits in series, as shown in Figure 3, where small capacitors link points 2 and 4 of one vibrator to points 3 and 1, respectively, of the next following vibrator. The resistors R1 and R2 and the capacitors C1 and C2 of each succeeding stage will be selected to give a suitably lower frequency than that of the preceding stage. During the blocked period of a_2, a small pip of voltage will be

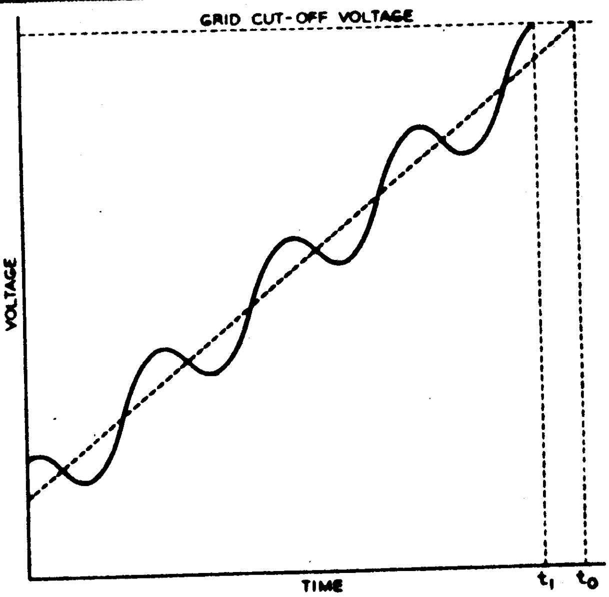

Fig. 2—Rising only because of flow of current through R1, the voltage follows the dashed line, but when an oscillator voltage is superimposed, the voltage follows the solid line

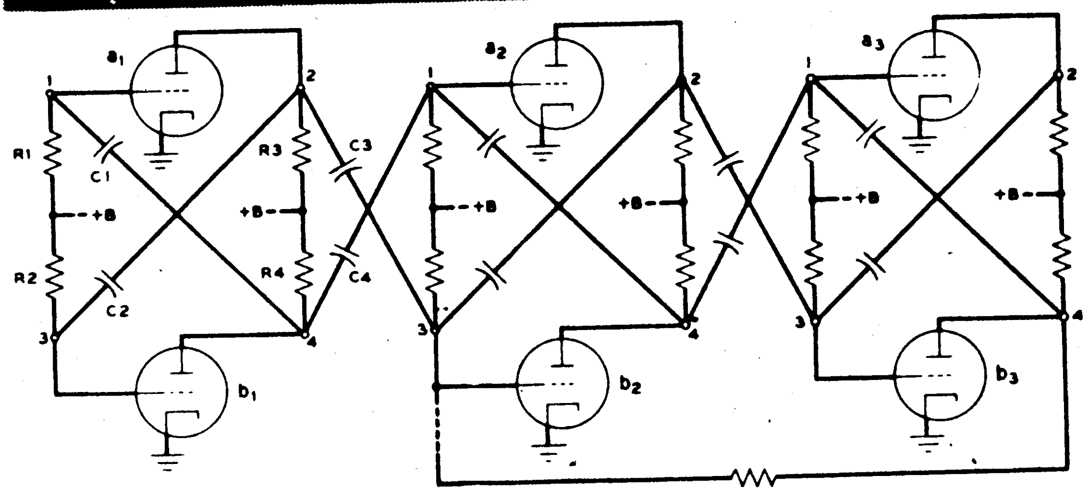

Fig. 3—Schematic of a three-stage multivibrator step-down

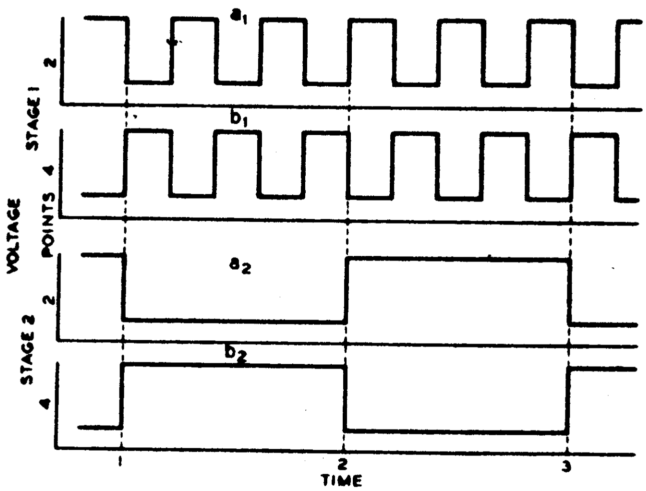

Fig. 4—Voltages across points 2 and 4 for the first few cycles of a multi-stage multivibrator step-down

superimposed on the rising voltage of its grid each time b_1 blocks, because of the sudden rise at this moment of the voltage at point 4 of the first stage. One of these pips will trigger the tube to pass current, much as do the positive waves of Figure 2. When a_2 starts to pass current as a result, b_2 will block, and it, in turn, will subsequently start to pass current after a fixed number of voltage pips from the c3 capacitor.

In Figure 4 are drawn the first few cycles of two stages of a multivibrator step-down. It is assumed that at TIME 1, a_1 starts to pass current and, in doing so, blocks b_1, causing a pip of voltage on the grid of a_2. It is assumed further that this is the pip that makes a_2 start to pass current and, in doing so, it will block b_2. At TIME 1, therefore, a_1 and a_2 start to pass current and b_1 and b_2 block. If both tubes of STAGE 2 trigger to passing on the third pip from the preceding stage, b_2 will start to pass at TIME 2, and thus block a_2 at the same time, and a_2 will subsequently start to pass current at TIME 3. The period of STAGE 2 is thus from TIME 1 to TIME 3, and comprises a blocked period for a_2 and a blocked period for b_2.

It may be seen from Figure 4 that the blocked period of b_2 includes three blocked periods of b_1 and two blocked periods of a_1, to which correspond two current-passing periods of b_1. Similarly, the blocked period of a_2 includes three blocked periods of a_1 and two of b_1. If the number of pips required to trigger b_2 to pass current is N_{b_2} and the number to trigger a_2 to passing is N_{a_2}, then—letting b represent the length of a blocked period—the lengths of the blocked periods of STAGE 2 may be expressed as:

$$B_{a_2} = N_{a_2} B_{a_1} + (N_{a_2} - 1) B_{b_1}$$
$$B_{b_2} = N_{b_2} B_{b_1} + (N_{a_2} - 1) B_{a_1}$$

The sum of these two blocked periods, which represents the period T_2 of the second stage, is thus:

$$T_2 = B_{a_2} + B_{b_2} = (N_{a_2} + N_{b_2} - 1)(B_{a_1} + B_{b_1}) = (N_{a_2} + N_{b_2} - 1)\, T_1$$

and the ratio of T_2 to T_1, R_2, is T_2 divided by T_1, and thus

$$R_2 = (N_{a_2} + N_{b_2} - 1)$$

A similar relationship holds between STAGE 3 and STAGE 2, and the over-all ratio is R_2 times R_3. If N were three throughout, the over-all ratio for the three stages would be 5×5=25. If a fourth stage with the same value of N were added, the over-all ratio would be 5×5×5=125, and so on for any number of stages. Since the over-all ratio is thus the product of several factors, it can never be a prime. The ratio obtainable in a single stage is limited by the difficulty in distinguishing between the heights of successive pips from the preceding stage when N becomes too large. The maximum ratio for one stage is usually limited to about 15, for reasons of stability, and thus by using one stage any prime up to 15—3, 5, 7, 11, 13—can be obtained, but above 13 no prime over-all ratio is obtainable.

Suppose, however, that a feedback circuit be run from point 4 of STAGE 3 to point 3 of STAGE 2, as indicated by the dotted line of Figure 3. During the blocked period of b_3, current will be fed over this connection to increase the rate at which the voltage at point 3 of STAGE 2 is rising. As a result, b_2 will require fewer pips from a_1 before it passes current. The result of this feedback is shown in dotted lines on Figure 5, on the basis that N_{b_2} with feedback is 1 instead of 3. The solid lines show the outputs of the various stages as they would have been without feedback.

While b_3 is passing current, the feedback voltage will drop so low as to have no effect, and thus the oscillation of STAGE 2 will follow one pattern while b_3 is blocked, and another while b_3 is passing. As a result, the half cycles of STAGE 3 will differ in length.

During the blocked period of a_3, while b_3 is passing current, there is no effect of feedback, and thus R_{a_3} will be the same as without feedback. During the blocked period of b_3, however, each blocked period of b_2 will be shortened, since it will endure for only one pip from a_1 instead of three. Since the pips caused by a_1 blocking are one period of STAGE 1 apart, each unit reduction in N_{b_2} reduces the length of the blocked period of b_2 by T_1. With a reduction of two in N_{b_2}, as in the example assumed, each blocked period of b_2 will be $2T_1$ shorter than before. In general, if δ represents the number of units by which N_{b_2} is shortened, each blocked period of b_2 becomes δT_1 shorter than without feedback. In a blocked period of b_3, however, there are N_{b_3} blocked periods of b_2, and thus with feedback the blocked period of b_3 is shortened by $N_{b_3}\delta T_1$. Without feedback, the length of a period of STAGE 3 was $R_2R_3T_1$ while with feedback, it will be $R_2R_3T_1 - N_{b_3}\delta T_1$, and the over-all ratio with feedback, T_3 divided by T_1, will be $R_2R_3 - N_{b_3}\delta$. For the figures assumed, the ratio without feedback is $5\times5=25$, while with feedback it is $(5\times5)-(3\times2)=19$, which is a prime. If an over-all ratio of 37 had been desired, which is also a prime, all the N's could have been made 4, and R_2 and R_3 would both be 7. Then, by making δ equal to 3, the over-all ratio would be $(7\times7)-(4\times3)=37$. In actual practice, it is generally preferable to keep δ as small as possible; in fact, it can be made equal to 1, and all integral ratios, prime or otherwise, can still be obtained by variations of the other factors.

Besides making a prime over-all ratio possible, feedback will also give a non-integral rational number. With feedback, the ratio of STAGE 3 to STAGE 2, R_3, is not changed, and thus the ratio of STAGE 2 is equal to the over-all ratio divided by R_3. Since with feedback the over-all ratio is $(R_2\times R_3)-N_{b_3}\delta$, the ratio of STAGE 2 is equal to this expression divided by R_3. If R_2' represents the value of R_2 when feedback is present, $R_2'=R_2-\frac{(N_{b_3}\times\delta)}{R_3}$. For the factors already used $R_2'=5-\frac{3\times2}{5}=3\ 4/5$,

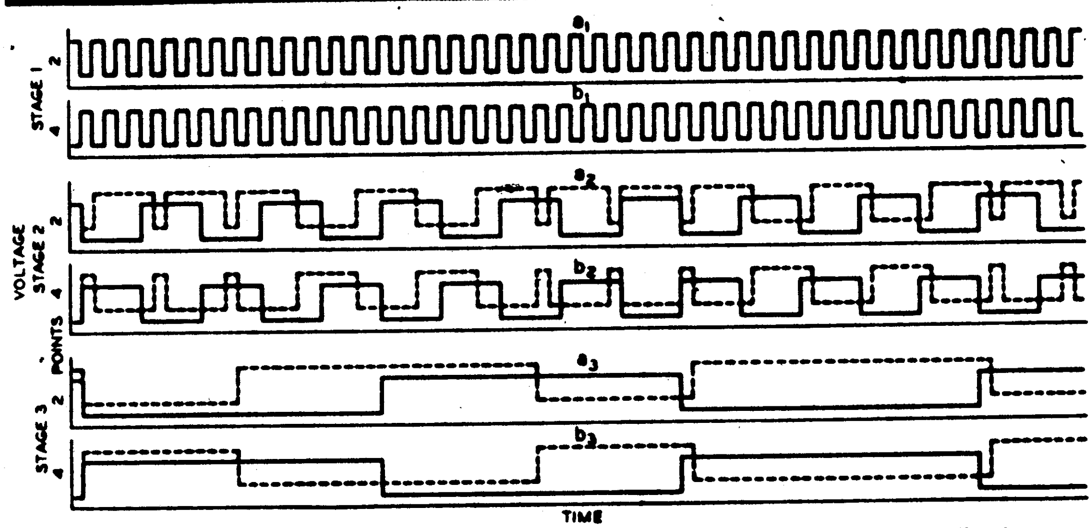

Fig. 5—Output voltages of a three-stage multivibrator circuit with and without feedback

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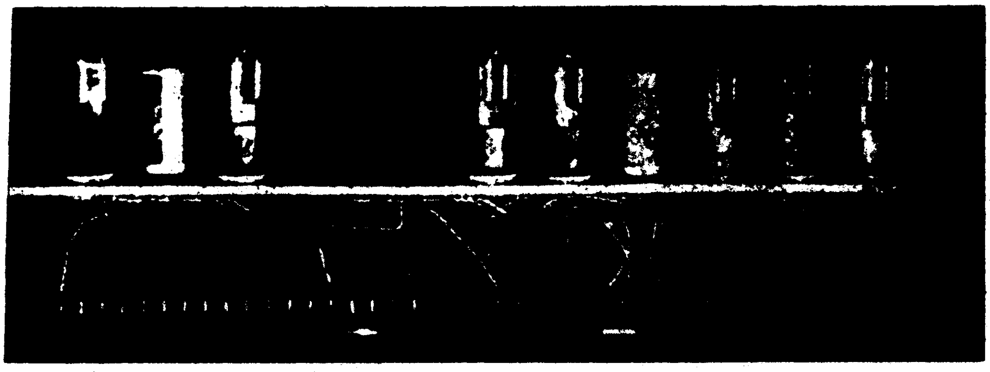

Fig. 6—The small space required is illustrated by the 2000-to-1, four-stage step-down mounted on the right half of the panel. Each tube includes two pentode units, and one tube serves as a driving unit and each of the other four as a step-down stage

and this non-integral ratio is obtained at the output of STAGE 2.

It will be noticed that the denominator of the fraction is R_3, and this fact indicates how any other ratio may be obtained. If a ratio of 4 3/7 were desired at the output of STAGE 2, for example, the equation would be $\frac{31}{7}=\frac{(7\times R_2)-N_{b_2}\delta}{7}$, and thus $7R_2-N_{b_2}\delta=$ 31. If R_2 is made 5, N_{a_2} made 6, N_{b_2} made 2, and δ is made 2, the result gives $(7\times5)-(2\times2)=31$, and R_2' would be $\frac{31}{7}=4\,3/7$.

On the assumption that no stage should have a greater ratio than 15, it would seem that no rational ratio with a denominator greater than 15 would be practicable. A ratio such as $14\frac{95}{121}$, for example, would appear unobtainable. If, however, a fourth stage is added and feedback is carried from the fourth to the second stage, such a ratio is readily obtainable at the output of the second stage. With such an arrangement, the over-all ratio is $R_2R_3R_4-\Delta$, where Δ represents the reduction in the length of B_{b_4} due to feedback. Since R_3 and R_4 are not changed by a feedback from the fourth to the second stage, the ratio at the output of the second stage is $\frac{R_2R_3R_4-\Delta}{R_3R_4}$. By making R_3 and R_4 each equal to 11, the denominator becomes the desired 121, and it then remains only to select the other parameters to secure the desired numerator.

When the feedback spans two stages, the calculation of the reduction in the length of T_4, called Δ in the above example, becomes a little more involved, but is found by similar reasoning. In the examples taken so far, N_2 has been equal to N_b, but this is not at all necessary, and for the more involved ratios unequal values may be needed. By taking advantage of such possibilities, and of the possibility of using as many stages as needed, with feedback spanning any group of them, a very wide range of non-integral or prime ratios is possible. The output waves are flat-topped, useful for many purposes, but where sine waves are desired, they are obtained by passing the output through a filter.

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