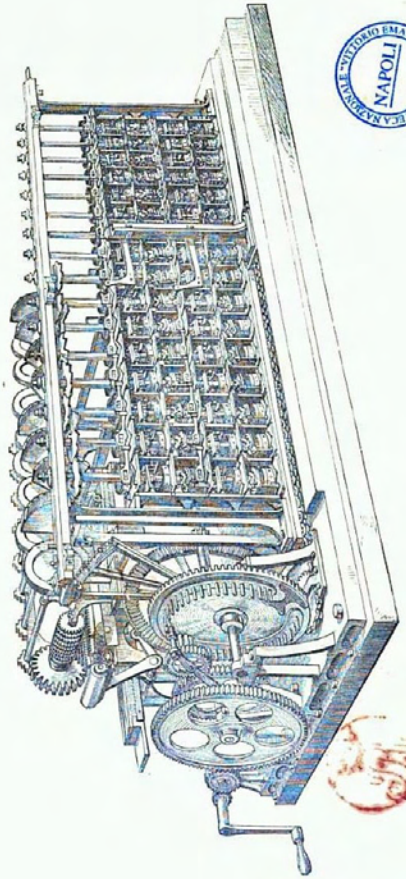
SPECIMEN TABLES

CALCULATED AND STEREOMOULDED

BY

THE SWEDISH

CALCULATING MACHINE.



SPECIMENS

OF

TABLES,

CALCULATED,

STEREOMOULDED, AND PRINTED

BY

MACHINERY.

LONDON:

LONGMAN, BROWN, GREEN, LONGMANS, AND ROBERTS,

PATERNOSTER ROW.

1857.

LONDON:
PRINTED (WITHOUT THE USE OF TYPES) BY C. WHITING, BEAUFORT HOUSE, STRAND.

TO

CHARLES BABBAGE, ESQ., F.R.S.,

&c. &c. &c.,

THESE PAGES ARE DEDICATED,

BY HIS SINCERE ADMIRERS,

GEORGE AND EDWARD SCHEUTZ.

PREFACE.

To those to whom this work is addressed it is unnecessary to point out the manifold advantages which have been derived in modern times from the various Numerical Tables now in the hands of the cultivators of science. The gigantic efforts that have led to the many important discoveries which have adorned the present century, would never have been made if these Tables had not existed. Astronomers, Navigators, Engineers, Actuaries, and indeed labourers in every department of science and the useful arts, could have made but little progress in their several vocations, had it not been for the resources supplied to them by those men of genius and of patience, who devoted their energies to this beneficial object: and when we contemplate the number and range of these Tables, and reflect upon the difficulties surmounted in their production, the time consumed, the labour employed, the capital expended, we cannot but regard with heartfelt gratitude and admiration those men who, without hesitation, and with an indomitable perseverance, devoted so many years of their lives to this irksome and herculean task; and that too, be it remembered, at a time when very few of their contemporaries could estimate the value of the work they were accomplishing. Such men may indeed be called the Martyrs of Science!

While acknowledging, however, the deep debt of gratitude which is due to the intellect, the industry, and the self-sacrifice of a Napier or a Briggs, we must still admit that, notwithstanding the pains taken to insure accuracy, much still remains to be done. There is scarcely any numerical table, or perhaps it may more truly be said that no one table has ever been published, which does not contain errors more or less numerous; and from a careful examination of a great number of existing tables, Mr. Babbage long ago arrived at the conviction that by

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employing the means hitherto resorted to for constructing them, it was impossible ever to obtain more than an approximate degree of correctness. He accordingly conceived the project of constructing an engine, which, by mechanism alone, would not only compute tabular numbers, but at the same time also impress its results upon plates of copper or other material, from which they might be immediately printed on paper, so as entirely to supersede the use of ordinary types.

This happy idea of combining the act of mechanical computation of series of numbers with that of simultaneously printing them, was perfectly original; although, when once propounded, it appears obviously to be the only mode by which, at a moderate cost, and with an infinite saving of labour, numerical tables can be obtained, with the positive certainty of their being wholly exempt from error.

Such was the origin of the *Calculating Machine*, invented by Mr. Babbage, and which, from its operating by the successive additions of several orders of differences derived from any numerical series, having all its terms connected by some analytical law, he denominated THE DIFFERENCE ENGINE.

The only analogous contrivances which had been effected, or even proposed, prior to Mr. Babbage's Difference Engine, were designed for the performance merely of single arithmetical operations of the most elementary kind, such as addition, subtraction, multiplication, and division—the very operations which numerical tables are intended to supersede, by sparing the inquirer the trouble of performing them either by the labour of the head or the manipulations of a machine, and presenting the results to his eye in the columns of a printed page.

The Difference Engine of Mr. Babbage had from time to time been cursorily noticed in several periodicals, when a circumstantial and elaborate disquisition on its merits and construction appeared in the *Edinburgh Review* for July, 1834. It was from the perusal of this article that Mr. George Scheutz, at that time the editor of a technological journal in Stockholm,

derived the first conception of constructing a machine for effecting the same purpose as that of Mr. Babbage, namely, that of calculating and simultaneously printing numerical tables.* But after he had satisfied himself of the practicability of the scheme, by constructing various models, composed of wood, pasteboard, and wire, he postponed to a future period the further prosecution of the design.

Three years afterwards, in the summer of 1837, his son, Mr. Edward Scheutz, at that time a student in the Royal Technological Institute at Stockholm, offered to his father to construct a working model in metal, on condition that he might have the use of a particular work-room in his father's house, as well as of a lathe and other necessary tools. With these implements he commenced the work, and during the summer vacation had so far completed it, as to demonstrate the feasibility of the scheme, and its applicability to the practical purposes for which it was designed.

Matters being so far advanced, his father made application to the Government on the 30th of October in the same year, for their sanction and assistance in the completion of the plan. To this application a negative answer was returned on the 21st of February in the following year.

Not discouraged by this refusal, Mr. E. Scheutz continued to employ his time on the model, holding frequent discussions with his father relative to the improvements in the original conception; sometimes the one and sometimes the other suggesting expedients for overcoming the difficulties that presented themselves. After many trials and many alterations, the calculating apparatus was, in the year 1840, so far completed, that it correctly calculated series with terms of five figures and one difference, also of five figures. On the 29th of April, 1842,

* The article in the *Edinburgh Review* above referred to also suggested to Mr. Alfred Deacon the contrivance of a difference engine for the same purpose, but on a plan of construction very different. This machine is still in existence, although not in a finished state; it was intended to calculate consecutive numbers to twenty figures, with three orders of differences; but, on account of the expense, printing the result was not contemplated.

the model was extended so as to calculate similar series with two and three orders of differences.

The following year, 1843, the printing apparatus, and all the other parts of the model, were in readiness for the inspection of the Royal Swedish Academy of Sciences. After several trials, a certificate was obtained on the 18th September, 1843, signed by Baron Jacob Berzelius, Secretary of the Academy; N. H. Selander, Astronomer of the Academy; and E. B. Lillieh̊ök, R. N., Professor of Natural Philosophy in the Royal Military Academy at Marieberg.

From this certificate the following passages are extracted :—

"The apparatus in question is composed of three parts.

"1st. The Calculating Machine.—It cannot compute series of a higher degree than the third, nor does it give complete terms exceeding five figures; but in the nature of the mechanism, there is nothing to prevent its extension to the working of series of any degree whatever, and to terms of as many figures as the purpose may require.

"2nd. The Printing Machine.—Every term given by the calculating apparatus is expressed by printed figures, closely arranged in lines, as in a printed table, the lines being impressed on some softer material, adapted to receive galvanoplastic or stereotyped copies. All the lines succeed each other very correctly in the same vertical column.

"3rd. The Numbering Machine.—With the printing machine, another apparatus is combined, which prints the arguments before every term. The machine is put in motion by turning the handle of a winch, by means of which, and without further manipulations, the calculation, as well as the printing and arranging of figures and lines, are effected."

The inventors sought for orders in various countries, making use of the above certificate as a recommendation; but, meeting with no success, the model remained shut up in its case during the ensuing seven years.

But in the year 1850, December 26th, another inspection was made by a committee of the Royal Academy of Sweden, consisting of General Baron Fabian Wrede, Chief of the Royal Military Academy at Marieberg; and the late P. A. Wallmark, R.N., Director of the Royal Technological Institute of Stockholm.

In the beginning of the following year, 1851, January 28, Mr. George Scheutz made a fresh application to the Government, to obtain from them the means of carrying the plan into full execution, by the construction of a larger and still more improved machine. His request was first referred to the judgment of the Academy of Sciences, which then advised the Government to consent to the application, because, as their Report says, "the Messrs. Scheutz had devoted many years of labour to this invention, which, without the least doubt, would be useful to science; and had incurred expenses, which, with relation to their resources, were very considerable; and that Mr. Scheutz, sen., had for a long time, without assistance from the public, published many useful works in his native country, relating to industrial progress."

The decision of the Government, given on the 29th April, was, that there were no public funds at their disposal for the object in question.

In the Diet of 1851, Mr A. M. Brinck, a merchant, and member of the Diet for Stockholm, moved that a national recompense should be given to the inventors. The motion was acceded to, on condition that the money should be paid to Messrs. G. and E. Scheutz after his Majesty the King had, on due examination, given it as his opinion that the machine was completed and answered its purpose. The inventors now sought to anticipate the reward, in order to have the means of completing the invention, and their petition was granted on the 24th October, 1851, on condition that the money was to be refunded if the machine was not completed before the end of the year 1853, or if, when completed, it was not found to answer its purpose.

In consequence of these limitations, the inventors were obliged, in the first instance, to procure a guarantee for the repayment, in case of failure. Partly by their own exertions, and partly by the kind assistance of some members of the

Swedish Academy, they were fortunate enough to collect a list of subscribers, each of whom rendered himself responsible for that part of the amount which was annexed to his name. The names of the subscribers are found in the subjoined note ;* and the author of this little book embraces this opportunity of expressing to them, in the names of the inventors of the machine, their gratitude for the means thus afforded of bringing it into its present improved state. Without such patronage, indeed, the machine would probably never have been completed, nor these tables produced.

The first working drawing of the new machine was finished on the 1st of February, 1852, and the machine itself was completed before the end of October, 1853.

The machine thus completed differs in many parts from the first model, which was confined in its range to the decimal scale, and could neither produce tables containing hours or degrees, minutes and seconds, nor effect the requisite corrections in the last printed figure. For the suggestion of these important improvements, the inventors are indebted to General Baron Fabian Wrede, who was one of the most zealous promoters of the undertaking. The machine was constructed in conformity with the drawings of Mr. Edward Scheutz, and under his immediate superintendence, at the manufactory of Mr. C. W. Bergström, at Stockholm; and when all the parts were finished, and finally put together, the new machine performed its work so perfectly, that the first trials required no alteration whatever. In the Report to H. M. the King of Sweden, the Royal Academy not only declared that the machine answered its purpose, but at the same time expressed its conviction, founded on the evidence of facts, that the expenditure for the completed machine had far exceeded the sum awarded as the national recompense. His Majesty then proposed to the Diet to give

* E. F. Berg	J. Falkenholm	P. W. Lundgren
C. F. Bergstedt	N. Grafström	B. O. Nycander
J. W. Bergström	L. J. Hierta	P. Siljeström
C. Crasselt	G. Lallerstedt	P. E. Swedbom
Eklund	Lovén	J. Wahlström

the inventors an additional recompense, of the same amount as the former one, which proposition was agreed to, the inventors receiving an official notice of this decision in their favour, from the Royal Academy of Sciences, on the 11th of August, 1854.

It was during the three last months of the same year, that the inventors visited England and France. On their arrival in London, they met with one of their countrymen, Count P. A. Sparre, by whom they were introduced to Messrs. Bryan Donkin & Co., Civil Engineers, who allowed the two foreigners to exhibit the machine at their manufactory in Bermondsey. It was here that they became acquainted with Mr. William Gravatt, F.R.S., Civil Engineer, who took a very warm interest in the invention, gave notice to the Royal Society of the arrival of the machine, and showed and explained it to several of the most eminent members of that learned body—namely, Colonel Edward Sabine, Doctor P. M. Roget, Professors Faraday, Hind, Stokes, Wheatstone, Miller, and several others.

After the departure of the inventors, the Royal Society, at Mr. Gravatt's request, most kindly allowed the machine to remain in one of their apartments in Somerset House, where Mr. Gravatt had also the kindness to work it, and to explain its mechanism on many occasions to the Fellows of that illustrious Society and other visitors. On the 29th of January, 1855, it was honoured by the inspection of His Royal Highness Prince Albert.

The machine was in the same year conveyed under the care of Count Sparre, to Paris, and placed in the Great Exhibition, where Mr. Gravatt again had the kindness to work and explain it to many scientific gentlemen. The jury to which it was referred, composed of distinguished persons of all nations, after a full examination, unanimously awarded the Gold Medal to the inventors, who received it publicly through the hands of His Royal Highness the Prince Charles, in the Royal Palace of Stockholm, on the 21st of April, 1856. Some time previously, Mr. Scheutz, sen., was elected a Member of the Swedish Academy of the Third Class for the Science of Mechanics; and

on the 28th of April he was made a Knight of the Order of Wasa.

A letter from Mr. Bryan Donkin, of the 5th June, 1856, induced Mr. E. Scheutz again to make a short visit to London, where he had the gratification of meeting those friends whose interest in the invention had remained unabated.

By the advice of Mr. Gravatt, to whom the inventors were already under such deep obligation, it was resolved that the machine should again be brought over to England, and, as a last attempt, that an endeavour should be made to obtain an acknowledgment of the practical usefulness of the invention by the publication of a collection of specimens of numerical Tables computed and printed by the machine. Mr. Gravatt had also the kindness to set apart a room in his own house for the performance of this work, and offered to promote the undertaking in every way. The inventor, now animated with fresh hopes, returned to France in order to fetch the machine, and it was during his stay there that he had the honour of showing the engine at work in the Imperial Observatory.

After Mr. E. Scheutz's return to London, as soon as the machine was put in order, and he had consulted Mr. Babbage and Mr. Gravatt with regard to the specimen Tables to be printed, this work was commenced, the calculations for setting the machine being made partly by Mr. Gravatt, according to formulæ of his own, and partly by Mr. E. Scheutz, after formulæ which General Baron Fabian Wrede had the kindness to give him on his departure from Sweden.

Meanwhile, what had been passing in London and in Paris had not escaped the notice of Professor B. A. Gould, of the Dudley Observatory, Albany, in the United States, who, conceiving the possibility of procuring purchasers for the machine in America, exerted himself in the most liberal and enlightened spirit to accomplish this object. The inventors had so many times seen their hopes blighted, that they did not expect that anything decisive would result from this new attempt; but they found themselves mistaken, for, after some time, they received information from Professor Gould that the terms they had proposed had been accepted; the machine, therefore, now belongs to the Dudley Observatory, at Albany, being a gift to that Observatory from an enlightened and public-spirited merchant of that city, John Fr. Rathbone, Esq.

In the course of their career, the inventors have had to go through many difficulties; but they have also been amply rewarded by the friendship and sympathy, which so many good and noble-minded men have manifested towards them, and by the certainty they now possess, that the object for which they have so long and so severely toiled, will not be lost. They believe, also, that if this limited collection of specimens should be deemed not sufficient to prove the practical usefulness of the invention, it will soon be followed by other proofs of greater importance, printed by the astronomers of the Western World; and that the individual to whom they have with gratitude and regard dedicated this little work, will then be further known for what he truly is—namely, one of the benefactors of mankind, and one among the noblest and most ingenious of the sons of England.

The drawing here given, is from an engraving kindly lent by the proprietor of the *Illustrated London News*, and exhibits the machine at a time when it has just completed the operation of stereotyping a result extending to eight places of figures.

It may be as well to premise that the size of the whole machine, when placed on its proper stand and protected by its cover, is about that of a small square pianoforte.

The calculating portion of the machine, which appears in the front of the drawing, consists of a series of fifteen upright steel axes, passing down the middle of five horizontal rows of silver-coated numbering rings, fifteen in each row, each ring being supported by, and turning concentrically on its own small brass shelf, having within it a hole rather less than the largest diameter of the ring. Round the cylindical surface of each ring are engraved the ordinary numerals from 0 to 9, one of which, in each position of the ring, appears in front, so that the successive numbers shown in any horizontal row of rings may be read from left to right, as in ordinary writing.

The upper row exhibits the number or answer resulting from

the calculation to fifteen places of figures, the first eight of which the machine stereotypes. The numbers seen on the second row of rings constitute the first order of differences, also to fifteen places of figures, if that number be required; and the third, fourth, and fifth row of rings, in like manner, exhibit the second, third, and fourth orders of differences. Any row can be set by hand, so as to present to the eye any number expressed according to the decimal scale of rotation; such as the number 987654321056789, the first eight figures of which, if in the uppermost row, would, on being calculated by the machine, be immediately stereotyped. But by simply changing a ring in each of two of the vertical columns, the machine can be made to exhibit and to calculate numbers expressed in the mixed senary system of notation, as in that of degrees, minutes, seconds, and decimals of a second. Thus, for instance, if the result 874324687356402 were indicated in the upper row of rings, it would be stereotyped 87 degrees 43 minutes 24.69 seconds. While this process is going on, the argument proper to each result is at the same time also stereotyped in its proper place; nothing more being required for that purpose than to set each row of figure rings to differences previously calculated from the proper formula, and to place a strip of sheet lead on the slide of the printing apparatus; then, by turning the handle (to do which requires no greater power than what is exerted in turning that of a small barrel-organ), the whole table required is calculated and *stereomoulded in the lead.* By this expression is meant that the strip of lead is made into a beautiful stereotype mould, from which any number of sharp stereotype plates can be produced ready for the working of an ordinary printing press. At the average rate of working the machine, 120 lines per hour of arguments and results are calculated and actually stereotyped, ready for the press. It is found on trial that the machine calculates and stereotypes, without chance of error, two-and-a-half pages of figures in the same time that a skilful compositor would take merely to set up the types for one single page.

The appearance of the tables would have been much improved had the machine been provided with better steel figure-wheels, the reason for not adopting which alteration may be gathered from the little historical sketch herein given.

In order to limit the size of this little book, the specimens given are neither so numerous nor so extensive as was at first intended. A great number of tables were suggested,* but they all so evidently came within the power of this machine, and as mere repetition was to be avoided, the short specimens Nos. 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 were considered sufficient as examples of tables perfectly easy with this machine, which happens to be limited to four orders of differences.

Specimen 12, the radius vector of Venus, and 13, the Sun's longitude for every twenty-four hours, are examples of calculations also easily within the power of four differences, but of which the number of terms obtainable at such wide intervals at one setting is considerably less than in the first-mentioned specimens. If, however, we wanted the Sun's longitude, or the radius vector of Venus, or any similar table, for intervals of every three hours for instance, it is evident that the machine could produce at each setting a table of eight times the extent of the specimens given.

In specimen 14, the logarithm of the Earth's radius vector, and still more in specimen 15, the logarithm of the distance between the Earth and Mars, in consequence of the wideness of the intervals, the want of more than four orders of difference, or the addition of some such contrivance as Mr. Babbage has alluded to in his letter to Sir H. Davy, July 3rd, 1823, begins to be felt. Should, however, five places of decimals be considered sufficient, double the number of terms could be produced in one setting in each of the specimens 14 and 15; or, if the intervals were smaller, the same remark would apply to these

* Such as Second Logarithms, Anti-logarithms, Powers of Sines, &c., Natural Sines, &c., Life Annuity Tables, Interest Tables, Difference of Latitude and Departure, Mensuration, Flow of Water in Canals and through Pipes, Earthwork Tables, &c.

specimens as to specimens 12 and 13; and it must be borne in mind, that, however short may be the range in this machine at any one setting, the answers are calculated indifferently in either the decimal or in the mixed sennary scales, and stereomoulded at the same time without chance of error.

The following is an abstract kindly furnished by Mr. Gravatt of his own manner of considering and of working this machine.

If u be any function of x, and we set in the machine its initial value $\underset{0}{u}$ and the finite differences $\Delta^1\underset{-1}{u}$ $\Delta^2\underset{-1}{u}$ $\Delta^3\underset{-2}{u}$ $\Delta^4\underset{-2}{u}$ in the order shown in the left-hand column below:—

$$
\begin{array}{lllll}
\underset{0}{u} & \underset{1}{u} & \underset{2}{u} & \underset{3}{u} & \ldots \text{ \&c.} \\
\Delta^1\underset{-1}{u} & \Delta^1\underset{0}{u} & \Delta^1\underset{1}{u} & \Delta^1\underset{2}{u} & \ldots \text{ \&c.} \\
\Delta^2\underset{-1}{u} & \Delta^2\underset{0}{u} & \Delta^2\underset{1}{u} & \Delta^2\underset{2}{u} & \ldots \text{ \&c.} \\
\Delta^3\underset{-2}{u} & \Delta^3\underset{-1}{u} & \Delta^3\underset{0}{u} & \Delta^3\underset{1}{u} & \ldots \text{ \&c.} \\
\Delta^4\underset{-2}{u} & \Delta^4\underset{-1}{u} & \Delta^4\underset{0}{u} & \Delta^4\underset{1}{u} & \ldots \text{ \&c.}
\end{array}
$$

Then in the first half-stroke the machine will simultaneously add the even differences to the odd differences immediately above them, giving the new odd differences $\Delta^3\underset{-1}{u}$ and $\Delta^1\underset{0}{u}$. At the next half-stroke it will add these new odd differences to the old even differences, thereby forming $\Delta^2\underset{0}{u}$ and $\underset{1}{u}$ and the machine will appear arranged as shown in the second column. In like manner will be formed the third, fourth, &c., columns, and if $\Delta^4\underset{-2}{u}=\Delta^4\underset{-1}{u}=\Delta^4\underset{0}{u}=$ &c., that is if the fourth difference is constant, the machine will go on calculating the successive values of u as long as we please to turn the handle.

We see, therefore, that this machine is at least capable of tabulating any function of x in which the fourth difference is constant.

As an example, let us take $\underset{0}{u}=1$ $\Delta^1\underset{-1}{u}=1$ $\Delta^2\underset{-1}{u}=2$ $\Delta^3\underset{-2}{u}=0$ $\Delta^4\underset{-2}{u}=0$; the machine when set would appear as in the left-hand column below, where the third and fourth differences being zero, are, for the sake of simplicity, omitted.

1	4	9	16	25	36 . . . &c.
1	3	5	7	9	11 . . . &c.
2	2	2	2	2	2 . . . &c.

Here at the first half-stroke 2 is added to 1 giving 3, and at the second half-stroke this 3 is added to 1 giving 4. Again, 2 is added to 3 giving 5, and this 5 is added to 4 giving 9, and so on; the machine as thus set producing the squares of the natural numbers for ever.

Again, if we set the machine to the numbers shown in the left-hand column below,

1	16	81	
1	15	65	175
14	50	110	
12	36	60	84
24	24	24	

24 will be added to 12 and *simultaneously* 14 to 1, giving respectively 36 and 15, and then 36 will be added to 14 and *simultaneously* 15 to 1, giving respectively 50 and 16, and so on; the machine as thus set producing the squares of the squares, or the fourth powers of the natural numbers as long as we please to turn the handle.

In the last example the machine is supposed to be stopped at a half-stroke, with only the *odd differences* 84 and 175 in the fourth column. Now, if we re-set the machine (changing the sign of these odd differences) as shown in the left-hand column below,

81	16	1	
−175	−65	−15	−1
110	50	14	
−84	−60	−36	−12
24	24	24	

we shall have $24 + -84 = -60$, and simultaneously $110 + -175 = -65$ for the first half-stroke, and $-60 + 110 = 50$, and simultaneously $-65 + 81 = 16$ for the second half-stroke, and so on. That is, if we in this way change the signs of the odd differences, the machine will, so to speak, go backwards, and that as long as we please to turn the handle.*

A negative number is expressed in the machine by its complement; thus, if to 56789, we wish to add, say minus 67 (that is, in fact, to subtract 67), we set the machine to add 99933 to 56789, thus :—

	56789		56789
less	67	added to	99933
gives	56722	gives	156722

but as in the machine there is purposely no figure-wheel to receive the left-hand 1, it is thrown (so to speak) into the air, and the machine shows and stereomoulds 56722 as it ought to do.

* In this way we can always make the machine itself show us the differences with which it was set.

This particular machine is capable of expressing u and its differences as far as the fourth order, to fifteen places of figures; but we may have to deal with differences of more than fifteen places, although we only require four, five, six, seven, or at most eight places of figures, to be printed as the tabular value of u.

Now if the machine be worked, so as to give very many values of u, it is evident that the omission of the 16th, &c., places of figures might begin to tell in the tabulated result.

To avoid this, it is necessary to know how many values of u we may obtain without an error, in the lowest figure of the printed result.

For this purpose, if we, considering what has gone before, imagine the machine set, as shown in the first column below, putting in only one value Δ, and that in the fourth order of differences,

1st	2d	3d	4th	5th	n^{th} Terms.
0	0	1Δ	5Δ	15Δ	$\ldots \frac{n-2}{1} \cdot \frac{n-1}{2} \cdot \frac{n}{3} \cdot \frac{n+1}{4} \Delta$
0	0	1Δ	4Δ	10Δ	$\ldots \frac{n-2}{1} \cdot \frac{n-1}{2} \cdot \frac{n}{3} \cdot \Delta$
0	1Δ	3Δ	6Δ	10Δ	$\ldots \frac{n-1}{1} \cdot \frac{n}{2} \cdot \Delta$
0	1Δ	2Δ	3Δ	4Δ	$\ldots \frac{n-1}{1} \cdot \Delta$
Δ	Δ	Δ	Δ	Δ	Δ

we should, in the manner in which the machine combines the differences (that is, always in *simultaneously formed couples*), obtain the 2nd, 3rd, 4th, 5th, and nth columns, the coefficients of Δ being necessarily and obviously *figurate numbers* of the various orders (in this case up to the 5th order) but with the 2nd and 3rd figurate numbers *set forward one term*, and the 4th and 5th figurate numbers *set forward two terms*—the law for any machine taking in any order of differences being evident. We immediately see that if we were to set in such a machine $\Delta^0 \Delta^1 \Delta^2 \Delta^3 \Delta^4$, putting P_n for the nth printed number we shall get

$$P_n = \Delta^0 + \frac{n-1}{1} \Delta^1 + \frac{n-1}{1} \frac{n}{2} \Delta^2 + \frac{n-2}{1} \frac{n-1}{2} \frac{n}{3} \Delta^3 + \frac{n-2}{1} \frac{n-1}{2} \frac{n}{3} \frac{n+1}{4} \Delta^4$$

Whence we see that if the error from leaving out the 16th, &c., figures, be α in the first difference, and β, γ, δ, respectively, in the 2nd, 3rd, and 4th differences (putting e the error in P_n), we should have e always less than $n\,\alpha + \frac{n^2}{2}\beta + \frac{n^3}{6}\gamma + \frac{n^4}{24}\delta$.

Now as we may always set the machine to $u_0 \pm \frac{1}{2} e$ instead of u_0, (thereby halving the effect of any error) we may, even when working with all the four differences, put practically $e = \frac{n^4}{48} \delta$.*

If m be the number of places required in the table, it is usual not to allow a greater error than ± 5 in the $m+1^{\text{th}}$ place. Now, the *greatest value* δ can have is (less than) $5 \times 10_{-10}$, whence we readily see that we may put $n^4 = 48 \times 10^{15-m}$. So that if, for instance, we required the machine to print 8 places correctly, we have $n^4 = 48 \times 10^7 = 480\ 000\ 000$ or $n = 148$ for the number of values of u that we can print in each direction.

If we wish to print only 7 places correctly, we have $n^4 = 48 \times 10^8$ or $n = 263$.

If we wish to print only 5 places correctly, we have $n^4 = 48 \times 10^{10}$ or $n = 832$, so that in this case we may print about 800 terms for each forward and for each backward working of the machine, or together about 1600 terms.

The following method of finding proper differences with which to set the machine (founded on the 5th lemma of the 3rd book of the Principia) is general, and extremely easy in practice. Let $u_x = u_0 + ax + bx^2 + cx^3 + dx^4$ now putting x successively $= 0 \pm 1$ and ± 2 we get

	1st Differences.	2nd Differences.
$u_{-2} = u_0 - 2a + 4b - 8c + 16d$		
	$a - 3b + 7c - 15d$	
$u_{-1} = u_0 - a + b - c + d$		$2b - 6c + 14d$
	$a - b + c - d$	
$u_0 = u_0$		$2b + 2d$
	$a + b + c + d$	
$u_1 = u_0 + a + b + c + d$		$2b + 6c + 14d$
	$a + 3b + 7c + 15d$	
$u_2 = u_0 + 2a + 4b + 8c + 16d$		

3rd Differences.	4th Difference.
$6c - 12d$	
	$24d$
$6c + 12d$	

* Instead of $\frac{n^4}{48}$ we really use $\frac{n^4 - 2n^3 - n^2 + 2n}{48}$

Whence $d = \frac{1}{24} \Delta^4 u_{-2}$ $\quad c = \frac{1}{2} \Delta^3 u_{-2} + 2d$ $\quad b = \frac{1}{2} \Delta^2 u_{-1} - d$ and $a = \Delta^1 u_{-1} + \frac{1}{2} \Delta^2 u_{-1} - c$.

Now, if $\text{N}-1$ be the number of terms to be interpolated between u_0 and $u_{\pm 1}$, $u_{\pm 1}$, and $u_{\pm 2}$, &c.; putting $\frac{1}{\text{N}} = x$ we get immediately

$$\Delta^4 u_{-2x} = 24dx^4$$
$$\Delta^3 u_{-2x} = 6(cx^3 - 2dx^4)$$
$$\Delta^2 u_{-x} = 2(bx^2 + dx^4)$$
$$\Delta^1 u_{-x} = ax - bx^2 + cx^3 - dx^4$$

With which differences we can set the machine and tabulate forwards from u_0 through u_1 to u_2, and by changing the sign of the odd differences in the manner before shown, we can tabulate backwards from u_{-3} through u_{-1} to u_{-2}, and this without knowing even the form of the function we are tabulating, it being sufficient that we have five values taken at equal intervals.*

Now u (the function of x, we may have to consider), may not have the 4th, nor indeed any order of differences constant, and to see the consequences of this let us compare (as in most cases will be amply sufficient) the value of the u_x derived from four orders of differences (which I write ${}^4u_x$) with the value of u_x derived from six orders of differences (which I write ${}^6u_x$).

In the equation $u_x = u_0 + a\,x^1 + b\,x^2 + c\,x^3 + d\,x^4 + e\,x^5 + f\,x^6$, putting x successively $0 + 1$, ± 2, ± 3, in the manner before shown, we get

$$f = \tfrac{1}{720} \Delta^6 u_{-3}$$
$$e = \tfrac{1}{120} \Delta^5 u_{-3} + \tfrac{1}{240} \Delta^6 u_{-3}$$
$$d = \tfrac{1}{24} \Delta^4 u_{-2} - \tfrac{1}{144} \Delta^6 u_{-3}$$
$$c = \tfrac{1}{6} \Delta^3 u_{-2} + \tfrac{1}{12} \Delta^4 u_{-2} - \tfrac{1}{24} \Delta^5 u_{-3} - \tfrac{1}{60} \Delta^6 u_{-3}$$
$$b = \tfrac{1}{2} \Delta^2 u_{-1} - \tfrac{1}{24} \Delta^4 u_{-2} + \tfrac{1}{100} \Delta^6 u_{-3}$$
$$a = \Delta^1 u_{-1} + \tfrac{1}{2} \Delta^2 u_{-1} - \tfrac{1}{6} \Delta^3 u_{-2} - \tfrac{1}{12} \Delta^4 u_{-2} + \tfrac{1}{30} \Delta^5 u_{-3} + \tfrac{1}{60} \Delta^6 u_{-3}$$

* The way I use the above formula in practice, is to begin, at the right-hand side of properly ruled paper, with the 4th differences, performing the calculations line by line in the order shown in the example given in page 8. In order to enable us to work always forwards from u_{-1} through u_0 to u_1 (which may sometimes be convenient) I have arranged a formula and given under it an example in page 9.

Now suppose we put

$$u_0 + ax + bx^2 + cx^3 + dx^4 + ex^5 + f x^6 = u_0 + \alpha x + \beta x^2 + \gamma x^3 + \delta x^4$$

rigidly only when $x = o \pm 1$ and ± 2, but within a certain degree of approximation only when x has any other values; we get directly

$$\delta = \tfrac{1}{24}\,\Delta^4 \underset{-2}{u} \qquad \gamma = \tfrac{1}{6}\,\Delta^3 \underset{-2}{u} + \tfrac{1}{12}\,\Delta^4 \underset{-2}{u} \qquad \beta = \tfrac{1}{2}\,\Delta^2 \underset{-1}{u} - \tfrac{1}{24}\,\Delta^4 \underset{-2}{u}$$

$$\alpha = \Delta^1 \underset{-1}{u} + \tfrac{1}{2}\,\Delta^2 \underset{-2}{u} - \tfrac{1}{6}\,\Delta^3 \underset{-2}{u} - \tfrac{1}{12}\,\Delta^4 \underset{-2}{u}$$

Whence

$$\begin{aligned}
{}^6u_x - {}^4u_x &= \tfrac{1}{720}\,\Delta^6 \underset{-3}{u}\, x^6 + \left(\tfrac{1}{120}\,\Delta^5 \underset{-3}{u} + \tfrac{1}{240}\,\Delta^6 \underset{-3}{u}\right) x^5 \\
&\quad - \tfrac{1}{144}\,\Delta^6 \underset{-3}{u}\, x^4 + \left(\tfrac{1}{24}\,\Delta^5 \underset{-3}{u} + \tfrac{1}{48}\,\Delta^6 \underset{-3}{u}\right) x^3 \\
&\quad + \tfrac{1}{180}\,\Delta^6 \underset{-3}{u}\, x^2 + \left(\tfrac{1}{30}\,\Delta^5 \underset{-3}{u} + \tfrac{1}{60}\,\Delta^6 \underset{-3}{u}\right) x \\
&= \tfrac{1}{720}\,\Delta^6 \underset{-3}{u} \left\{x^6 + 3\,x^5 - 5\,x^4 - 15\,x^3 + 4\,x^2 + 12\,x\right\} + \\
&\quad \tfrac{1}{120}\,\Delta^5 \underset{-3}{u} \left\{x^5 - 5\,x^3 + 4\,x\right\}
\end{aligned}$$

Now if we attend to the spirit of Newton's lemma, we see that the errors arising from the use of four differences, when we ought to use six differences, must be nearly maximums when $x = \pm\frac{1}{2} \pm \frac{3}{2}$, and putting in our equation these values of x, we have rigidly

$$\begin{aligned}
{}^6\underset{\frac{1}{2}}{u} - {}^4\underset{\frac{1}{2}}{u} &= \tfrac{3}{256}\,\Delta^5 \underset{-3}{u} + \tfrac{7}{1024}\,\Delta^6 \underset{-3}{u} \\
{}^6\underset{-\frac{1}{2}}{u} - {}^4\underset{-\frac{1}{2}}{u} &= -\tfrac{3}{256}\,\Delta^5 \underset{-3}{u} - \tfrac{5}{1024}\,\Delta^6 \underset{-3}{u} \\
{}^6\underset{\frac{3}{2}}{u} - {}^4\underset{\frac{3}{2}}{u} &= -\tfrac{7}{256}\,\Delta^5 \underset{-3}{u} - \tfrac{21}{1024}\,\Delta^6 \underset{-3}{u} \\
{}^6\underset{-\frac{3}{2}}{u} - {}^4\underset{-\frac{3}{2}}{u} &= \tfrac{7}{256}\,\Delta^5 \underset{-3}{u} + \tfrac{7}{1024}\,\Delta^6 \underset{-3}{u}
\end{aligned}$$

Or, in practice, the maximum *half error* may be taken as

$$.001\ (6\ \Delta^5 \underset{-3}{u} + 3\tfrac{1}{2}.\Delta^6 \underset{-3}{u}\ ) \text{ between } \underset{0}{u} \text{ and } \underset{1}{u}$$

$$.001\ (-6\Delta^5 \underset{-3}{u} - 2\tfrac{1}{2}.\Delta^6 \underset{-3}{u}\ ) \text{ between } \underset{0}{u} \text{ and } \underset{-1}{u}$$

$$.001\ (-14\Delta^5 \underset{-3}{u} - 10\tfrac{1}{2}.\Delta^6 \underset{-3}{u}\ ) \text{ between } \underset{1}{u} \text{ and } \underset{2}{u}$$

$$.001\ (14\Delta^5 \underset{-3}{u} + 3\tfrac{1}{2}.\Delta^6 \underset{-3}{u}\ ) \text{ between } \underset{-1}{u} \text{ and } \underset{-2}{u}$$

and these errors will of course be kept out of our tables; that is, we shall use one place of figures less than that which would be affected.

As an easy (and certainly a very favourable) example of the power of the machine, let us with it calculate a table of the Logarithms of the natural numbers up to 10,000, to five places of decimals. We must first actually calculate by any of the known methods the logarithms of 2, 3, 7, 11, 17, and 19, to seven places of decimals, from which we shall immediately obtain the logarithms of

10	20	40	80
11	22	44	84
12	24	48	88
13	26	52	92
14	28	56	96
15	30	60	100
16	32	64	
17	34	68	
18	36	72	
19	38	76	

To which, applying the formulæ just given, we, by ten easy calculations,* and by ten forward and ten backward settings of the machine, shall obtain a stereomoulded table up to 10,000.

The time occupied by the machine at its ordinary rate of working (namely, 120 numbers per hour), would be seventy-five hours; the ten plus and minus (say twenty) settings would not take up quite two hours; the calculations by the formulæ given of the proper differences to be set in the machine, could not cause a delay of two hours, the time taken (exclusive of the preliminary calculation of the logarithms of 2, 3, 7, 11, 17, and 19) being altogether about seventy-nine, or say eighty hours.

* See next page.

$$\begin{cases} \Delta^1 u + \frac{1}{2}\Delta^2 u - c = a \\ a\,\underset{-1}{x} - b\,x^2 + c\,x^3 - d\,x^4 = \Delta^1 u_{\underset{-1}{x}} \end{cases} \quad \begin{cases} \Delta^2 u \,.\, \frac{1}{2} - d = b \\ b\,\underset{-2}{x^2}.\,2 + 2\,d\,x^4 = \Delta^2_{\underset{-2}{x}} \end{cases} \quad \begin{cases} (\Delta^3 u + \frac{1}{2}\Delta^4 u)\,\frac{1}{6} = c \\ (c\,.\,\underset{-2}{x^3} - 2\,d\,\underset{-2}{x^4})\,.\,6 = \Delta^3 u_{\underset{-2}{x}} \end{cases} \quad \begin{cases} \Delta^4 u \,.\, \frac{1}{4} \cdot \frac{1}{12} = d \\ d\,.\,\underset{-2}{x^4}\,.\,2\,.\,12 = \Delta^4 u_{\underset{-2}{x}} \end{cases}$$

As an example, take $u_0 = \log.\ 3000 = \cdot 4771213$; $N = 200$. On ruled paper the calculation would appear as below:

```
1st Diff. x = 10^-2 ÷ 2                2nd Diff. x^2 = 10^-4 ÷ 4
          +                                        -
    2996 33;                               1934 6;
      96 73;                                967 3;
    2899 60;                                  21 1; 66667 = d
        4 34; 666667 = c                     965 1; 33333 = b
    2895 23; 333333 = a                  ; 2412 83333 = bx²
    1447; 62 666667 = ax                 ; 4825 66666
          ; 24 128333 = bx²              ;          27 = 2dx⁴
          ;       5433 = cx³             ; 4825 66693
          ;          1 = dx⁴             ·99; 5174 3330,7 = Δ²
·0001447; 86 800434, = Δ¹
·0001447 38 543765
9998552 61 456235 = −Δ¹

3rd Diff. x = 10^-6 ÷ 8                4th Diff. x^4 = 10^-8 ÷ (4 × 4)
          +                                        -
    2868;                                  520;
      260;                                 260;
    2608;                                    21; 667 = d
      434; 6667 = c                      ;     5417
;     54 3333 = cx³                      ;     1354 = dx⁴
;        271 = 2dx⁴                      ;     2708 = 2dx⁴
;     54 3604                            ;     32496
·00; 000326 16,24 = Δ³                   ·99; 999999967, 504 = Δ⁴

00; 000325 83
·99; 999674 17 = −Δ³
```

From which we get Logs. 2600, 2601....3400 or 800 Logarithms.

The calculation for the 1st Diff. is extended five places of decimals further than is necessary, in order to show how cx^3 and dx^4 would come in if they were required, and for a similar reason the calculations for the 2nd and 3rd Diffs. are extended respectively four and three places. The semicolon here marks off the 7th place of decimals, and the comma the 15th or last figure in the machine.

1st Diff.		2nd Diff.	3rd Diff.		4th Diff.
$\left(\Delta^1_{-1} - \frac{1}{2}\Delta^2_{-1}\right)\frac{1}{N}$		$\Delta^2_{-1}\frac{1}{N^2}$	$\left(\Delta^3_{-2} - \frac{1}{2}\Delta^4_{-2}\right)\frac{1}{N^3}$		$\Delta^4_{-2}\frac{1}{N^4}$
$-\ \Delta^2_{-1}\left(\frac{1}{N} + \frac{1}{2N^2}\right)$	(F)	$-\left(\Delta^3_{-2} - \frac{1}{2}\Delta^4_{-2}\right)\frac{1}{N}$	$-\ \Delta^4_{-2}\left(\frac{1}{N^3} + \frac{1}{2N^4}\right.$	(A)	
$+\left(\Delta^3_{-2} - \frac{1}{2}\Delta^4_{-2}\right)\left(\frac{1}{N^2} + \frac{1}{2N^3}\right)\frac{N+1}{3}$	(E)	$+\left(5\Delta^4_{-2}\frac{1}{N^2} + \Delta^4_{-2}\frac{1}{N^3}\right)\frac{1}{12}$			
$-\ \Delta^4_{-2}\left(\frac{1}{N^3} + \frac{1}{2N^4}\right)\frac{N+1}{3}\frac{N+1}{4}$	(C)				

As an example let $u = 35^\circ\ 38'\ 3''\cdot 71$, the Sun's Longitude March 13th, 1858, to tabulate the Longitude for March 13th, 14th, 15th....25th. Putting N = 6, we have as below :—

$u_{-1} = 35^\circ\ 2'\ 38'\ 3''\cdot 71$

```
1st Diff.                  2nd Diff.                  3rd Diff.                  4th Diff.
          +                          -                          -                          +
  2148 8·500 = Δ¹₁         80·2000 = Δ²₁            12·20000 = Δ³₁            12·00000 = Δ⁴₁
     40·1                  13·3667                   6·0                       2·0
  2158 8·6                  2·2278                  18·2                        ·533333
   3588·100                  ·5056 D                 3·03333                    ·055556
     1448 1F                 1·7222                   ·50556 D                 0·009259 = Δ⁴
   3602·581                  ·1397 B                  ·08426
      1·278 E                1·5825                   ·06019 A                   004630
   3601·303                                           ·14445                    ·035556
      ·246 C                99  59  58·4175 = Δ²      99  59  59·83555 = Δ³     ·060186 A
   3601·057
  0  0  1·057 = Δ¹
                            − 13·3667                                           333333
                            −  1·1139                                          1·666666
                            − 14·4806 F                 ·50556 D               ·009259
                                                        ·04213                 1·675925
                                                        ·54769                  ·139660 B
                                                       3·83383
                                                       1·27794 E               ·060185 A
                                                                               ·421295
                                                                              2·949065
                                                                               ·245755 C
```

Giving Sun's Longitude, March 13th to 25th, 1858.

No. I.

LOGARITHMS

OF

NUMBERS

FROM

1 TO 10,000,

CALCULATED, STEREOMOULDED, AND PRINTED

BY

MACHINERY.

1000	00000	1050	02119	1100	04139	1150	06070	1200	07918	1250	09691
1001	00043	1051	02160	1101	04179	1151	06108	1201	07954	1251	09726
1002	00087	1052	02202	1102	04218	1152	06145	1202	07990	1252	09760
1003	00130	1053	02243	1103	04258	1153	06183	1203	08027	1253	09795
1004	00173	1054	02284	1104	04297	1154	06221	1204	08063	1254	09830
1005	00217	1055	02325	1105	04336	1155	06258	1205	08099	1255	09864
1006	00260	1056	02366	1106	04376	1156	06296	1206	08135	1256	09899
1007	00303	1057	02407	1107	04415	1157	06333	1207	08171	1257	09934
1008	00346	1058	02449	1108	04454	1158	06371	1208	08207	1258	09968
1009	00389	1059	02490	1109	04493	1159	06408	1209	08243	1259	10003
1010	00432	1060	02531	1110	04532	1160	06446	1210	08279	1260	10037
1011	00475	1061	02572	1111	04571	1161	06483	1211	08314	1261	10072
1012	00518	1062	02612	1112	04610	1162	06521	1212	08350	1262	10106
1013	00561	1063	02653	1113	04650	1163	06558	1213	08386	1263	10140
1014	00604	1064	02694	1114	04689	1164	06595	1214	08422	1264	10175
1015	00647	1065	02735	1115	04727	1165	06633	1215	08458	1265	10209
1016	00689	1066	02776	1116	04766	1166	06670	1216	08493	1266	10243
1017	00732	1067	02816	1117	04805	1167	06707	1217	08529	1267	10278
1018	00775	1068	02857	1118	04844	1168	06744	1218	08565	1268	10312
1019	00817	1069	02898	1119	04883	1169	06781	1219	08600	1269	10346
1020	00860	1070	02938	1120	04922	1170	06819	1220	08636	1270	10380
1021	00903	1071	02979	1121	04961	1171	06856	1221	08672	1271	10415
1022	00945	1072	03019	1122	04999	1172	06893	1222	08707	1272	10449
1023	00988	1073	03060	1123	05038	1173	06930	1223	08743	1273	10483
1024	01030	1074	03100	1124	05077	1174	06967	1224	08778	1274	10517
1025	01072	1075	03141	1125	05115	1175	07004	1225	08814	1275	10551
1026	01115	1076	03181	1126	05154	1176	07041	1226	08849	1276	10585
1027	01157	1077	03222	1127	05192	1177	07078	1227	08884	1277	10619
1028	01199	1078	03262	1128	05231	1178	07115	1228	08920	1278	10653
1029	01242	1079	03302	1129	05269	1179	07151	1229	08955	1279	10687
1030	01284	1080	03342	1130	05308	1180	07188	1230	08991	1280	10721
1031	01326	1081	03383	1131	05346	1181	07225	1231	09026	1281	10755
1032	01368	1082	03423	1132	05385	1182	07262	1232	09061	1282	10789
1033	01410	1083	03463	1133	05423	1183	07298	1233	09096	1283	10823
1034	01452	1084	03503	1134	05461	1184	07335	1234	09132	1284	10857
1035	01494	1085	03543	1135	05500	1185	07372	1235	09167	1285	10890
1036	01536	1086	03583	1136	05538	1186	07408	1236	09202	1286	10924
1037	01578	1087	03623	1137	05576	1187	07445	1237	09237	1287	10958
1038	01620	1088	03663	1138	05614	1188	07482	1238	09272	1288	10992
1039	01662	1089	03703	1139	05652	1189	07518	1239	09307	1289	11025
1040	01703	1090	03743	1140	05690	1190	07555	1240	09342	1290	11059
1041	01745	1091	03782	1141	05729	1191	07591	1241	09377	1291	11093
1042	01787	1092	03822	1142	05767	1192	07628	1242	09412	1292	11126
1043	01828	1093	03862	1143	05805	1193	07664	1243	09447	1293	11160
1044	01870	1094	03902	1144	05843	1194	07700	1244	09482	1294	11193
1045	01912	1095	03941	1145	05881	1195	07737	1245	09517	1295	11227
1046	01953	1096	03981	1146	05918	1196	07773	1246	09552	1296	11260
1047	01995	1097	04021	1147	05956	1197	07809	1247	09587	1297	11294
1048	02036	1098	04060	1148	05994	1198	07846	1248	09621	1298	11327
1049	02078	1099	04100	1149	06032	1199	07882	1249	09656	1299	11361
1050	02119	1100	04139	1150	06070	1200	07918	1250	09691	1300	11394

1300	11394	1350	13033	1400	14613	1450	16137	1500	17609
1301	11428	1351	13066	1401	14644	1451	16167	1501	17638
1302	11461	1352	13098	1402	14675	1452	16197	1502	17667
1303	11494	1353	13130	1403	14706	1453	16227	1503	17696
1304	11528	1354	13162	1404	14737	1454	16256	1504	17725
1305	11561	1355	13194	1405	14768	1455	16286	1505	17754
1306	11594	1356	13226	1406	14799	1456	16316	1506	17783
1307	11628	1357	13258	1407	14829	1457	16346	1507	17811
1308	11661	1358	13290	1408	14860	1458	16376	1508	17840
1309	11694	1359	13322	1409	14891	1459	16406	1509	17869
1310	11727	1360	13354	1410	14922	1460	16435	1510	17898
1311	11760	1361	13386	1411	14953	1461	16465	1511	17926
1312	11793	1362	13418	1412	14983	1462	16495	1512	17955
1313	11826	1363	13450	1413	15014	1463	16524	1513	17984
1314	11860	1364	13481	1414	15045	1464	16554	1514	18013
1315	11893	1365	13513	1415	15076	1465	16584	1515	18041
1316	11926	1366	13545	1416	15106	1466	16613	1516	18070
1317	11959	1367	13577	1417	15137	1467	16643	1517	18099
1318	11992	1368	13609	1418	15168	1468	16673	1518	18127
1319	12024	1369	13640	1419	15198	1469	16702	1519	18156
1320	12057	1370	13672	1420	15229	1470	16732	1520	18184
1321	12090	1371	13704	1421	15259	1471	16761	1521	18213
1322	12123	1372	13735	1422	15290	1472	16791	1522	18241
1323	12156	1373	13767	1423	15320	1473	16820	1523	18270
1324	12189	1374	13799	1424	15351	1474	16850	1524	18298
1325	12222	1375	13830	1425	15381	1475	16879	1525	18327
1326	12254	1376	13862	1426	15412	1476	16909	1526	18355
1327	12287	1377	13893	1427	15442	1477	16938	1527	18384
1328	12320	1378	13925	1428	15473	1478	16967	1528	18412
1329	12352	1379	13956	1429	15503	1479	16997	1529	18441
1330	12385	1380	13988	1430	15534	1480	17026	1530	18469
1331	12418	1381	14019	1431	15564	1481	17056	1531	18498
1332	12450	1382	14051	1432	15594	1482	17085	1532	18526
1333	12483	1383	14082	1433	15625	1483	17114	1533	18554
1334	12516	1384	14114	1434	15655	1484	17143	1534	18583
1335	12548	1385	14145	1435	15685	1485	17173	1535	18611
1336	12581	1386	14176	1436	15715	1486	17202	1536	18639
1337	12613	1387	14208	1437	15746	1487	17231	1537	18667
1338	12646	1388	14239	1438	15776	1488	17260	1538	18696
1339	12678	1389	14270	1439	15806	1489	17289	1539	18724
1340	12710	1390	14301	1440	15836	1490	17319	1540	18752
1341	12743	1391	14333	1441	15866	1491	17348	1541	18780
1342	12775	1392	14364	1442	15897	1492	17377	1542	18808
1343	12808	1393	14395	1443	15927	1493	17406	1543	18837
1344	12840	1394	14426	1444	15957	1494	17435	1544	18865
1345	12872	1395	14457	1445	15987	1495	17464	1545	18893
1346	12905	1396	14489	1446	16017	1496	17493	1546	18921
1347	12937	1397	14520	1447	16047	1497	17522	1547	18949
1348	12969	1398	14551	1448	16077	1498	17551	1548	18977
1349	13001	1399	14582	1449	16107	1499	17580	1549	19005
1350	13033	1400	14613	1450	16137	1500	17609	1550	19033

1600	20412	1650	21748	1700	23045	1750	24304	1800	25527
1601	20439	1651	21775	1701	23070	1751	24329	1801	25551
1602	20466	1652	21801	1702	23096	1752	24353	1802	25575
1603	20493	1653	21827	1703	23121	1753	24378	1803	25600
1604	20520	1654	21854	1704	23147	1754	24403	1804	25624
1605	20548	1655	21880	1705	23172	1755	24428	1805	25648
1606	20575	1656	21906	1706	23198	1756	24452	1806	25672
1607	20602	1657	21932	1707	23223	1757	24477	1807	25696
1608	20629	1658	21958	1708	23249	1758	24502	1808	25720
1609	20656	1659	21985	1709	23274	1759	24527	1809	25744
1610	20683	1660	22011	1710	23300	1760	24551	1810	25768
1611	20710	1661	22037	1711	23325	1761	24576	1811	25792
1612	20737	1662	22063	1712	23350	1762	24601	1812	25816
1613	20763	1663	22089	1713	23376	1763	24625	1813	25840
1614	20790	1664	22115	1714	23401	1764	24650	1814	25864
1615	20817	1665	22141	1715	23426	1765	24674	1815	25888
1616	20844	1666	22167	1716	23452	1766	24699	1816	25912
1617	20871	1667	22194	1717	23477	1767	24724	1817	25935
1618	20898	1668	22220	1718	23502	1768	24748	1818	25959
1619	20925	1669	22246	1719	23528	1769	24773	1819	25983
1620	20952	1670	22272	1720	23553	1770	24797	1820	26007
1621	20978	1671	22298	1721	23578	1771	24822	1821	26031
1622	21005	1672	22324	1722	23603	1772	24846	1822	26055
1623	21032	1673	22350	1723	23629	1773	24871	1823	26079
1624	21059	1674	22376	1724	23654	1774	24895	1824	26102
1625	21085	1675	22401	1725	23679	1775	24920	1825	26126
1626	21112	1676	22427	1726	23704	1776	24944	1826	26150
1627	21139	1677	22453	1727	23729	1777	24969	1827	26174
1628	21165	1678	22479	1728	23754	1778	24993	1828	26198
1629	21192	1679	22505	1729	23779	1779	25018	1829	26221
1630	21219	1680	22531	1730	23805	1780	25042	1830	26245
1631	21245	1681	22557	1731	23830	1781	25066	1831	26269
1632	21272	1682	22583	1732	23855	1782	25091	1832	26293
1633	21299	1683	22608	1733	23880	1783	25115	1833	26316
1634	21325	1684	22634	1734	23905	1784	25139	1834	26340
1635	21352	1685	22660	1735	23930	1785	25164	1835	26364
1636	21378	1686	22686	1736	23955	1786	25188	1836	26387
1637	21405	1687	22712	1737	23980	1787	25212	1837	26411
1638	21431	1688	22737	1738	24005	1788	25237	1838	26435
1639	21458	1689	22763	1739	24030	1789	25261	1839	26458
1640	21484	1690	22789	1740	24055	1790	25285	1840	26482
1641	21511	1691	22814	1741	24080	1791	25310	1841	26505
1642	21537	1692	22840	1742	24105	1792	25334	1842	26529
1643	21564	1693	22866	1743	24130	1793	25358	1843	26553
1644	21590	1694	22891	1744	24155	1794	25382	1844	26576
1645	21617	1695	22917	1745	24180	1795	25406	1845	26600
1646	21643	1696	22943	1746	24204	1796	25431	1846	26623
1647	21669	1697	22968	1747	24229	1797	25455	1847	26647
1648	21696	1698	22994	1748	24254	1798	25479	1848	26670
1649	21722	1699	23019	1749	24279	1799	25503	1849	26694
1650	21748	1700	23045	1750	24304	1800	25527	1850	26717