

Dick Richards

Boolean Algebra

$$0+0=0$$

$$0 \times 0 = 0$$

$$0+1=1$$

$$0 \times 1 = 0$$

$$1+1=1$$

$$1 \times 1 = 1$$

or

and

$$A+0=A$$

$$A \cdot 0 = 0$$

$$A+1=1$$

$$A \cdot 1 = A$$

$$A+A=A$$

$$A \cdot A = A$$

Commutative

$$A+B = B+A$$

$$AB = BA$$

Associative

$$(A+B)+C = A+(B+C)$$

$$(AB)C = A(BC)$$

$$AB+AC = A(B+C)$$

$$A+B+C = (A+B)(A+C)$$

$$A(A+B)=A$$

$$\bar{1} = 0$$

$$\overline{A+B+C} = \bar{A}\bar{B}\bar{C}$$

$$\bar{0} = 1$$

$$\overline{ABC + BCA + CAB} = \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C}\bar{A} + \bar{C}\bar{A}\bar{B}$$

$$A+\bar{A}=1$$

$$\overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}\bar{B} + \bar{B}\bar{C}\bar{A}} = (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{C}+\bar{A})$$

$$A\bar{A}=0$$

$$= (\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{A} + \bar{B}\bar{C} + \bar{C}\bar{A})$$

$$\bar{A} = A$$

$$= (\bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}\bar{B} + \bar{B}\bar{C}\bar{A})$$

$$\bar{A}+\bar{B}+\bar{C}=\overline{ABC}$$

$$= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}$$

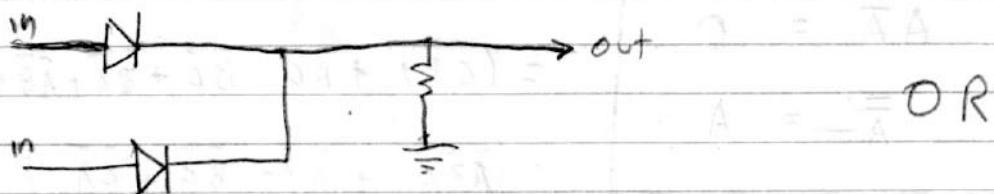
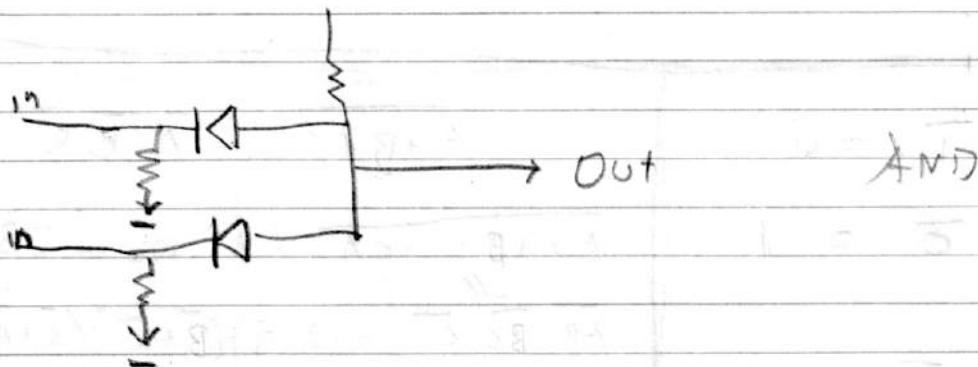
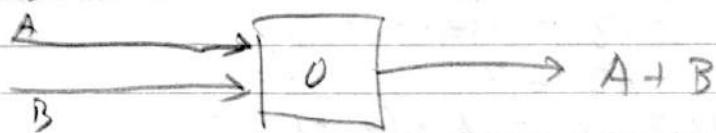
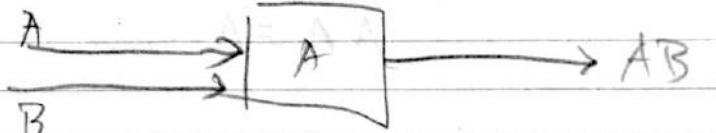
more or less

$$(A\bar{B} + B\bar{C} + C\bar{A}) (\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}) = (A+B+C)(\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A})$$

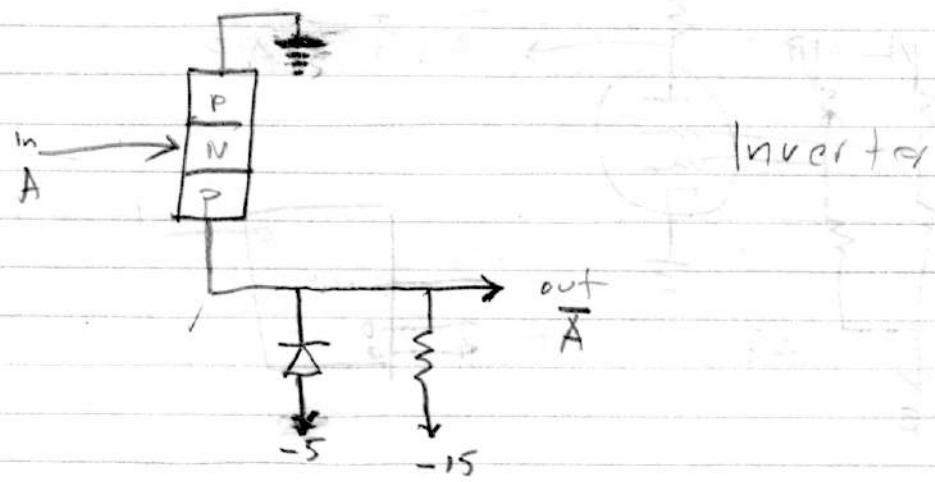
\uparrow_{prove}

Note that

$$A\bar{B} + B\bar{C} + C\bar{A} \neq A + B + C \quad \leftarrow \text{prove}$$

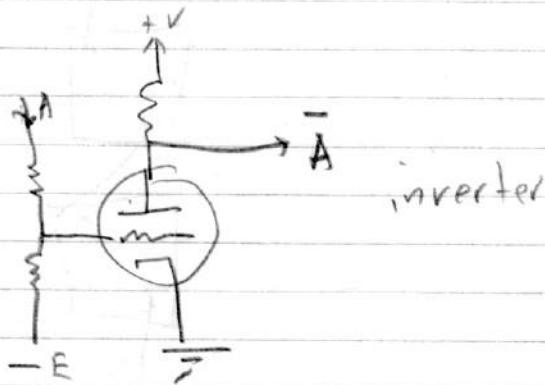


7/2 c

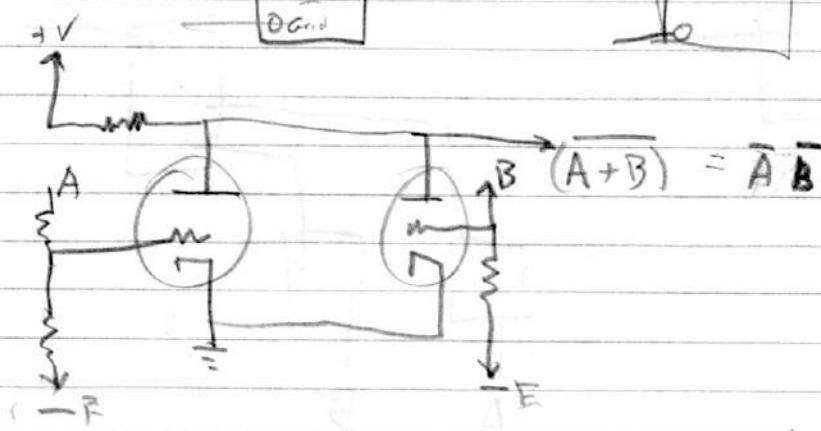
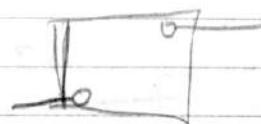
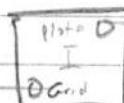


Prove

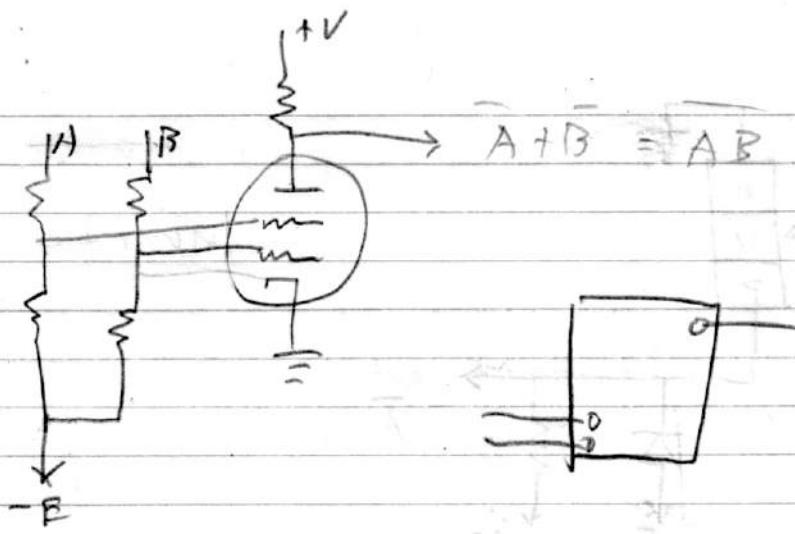
$$AB + CD = (A+C)(A+D)(B+C)(B+D)$$



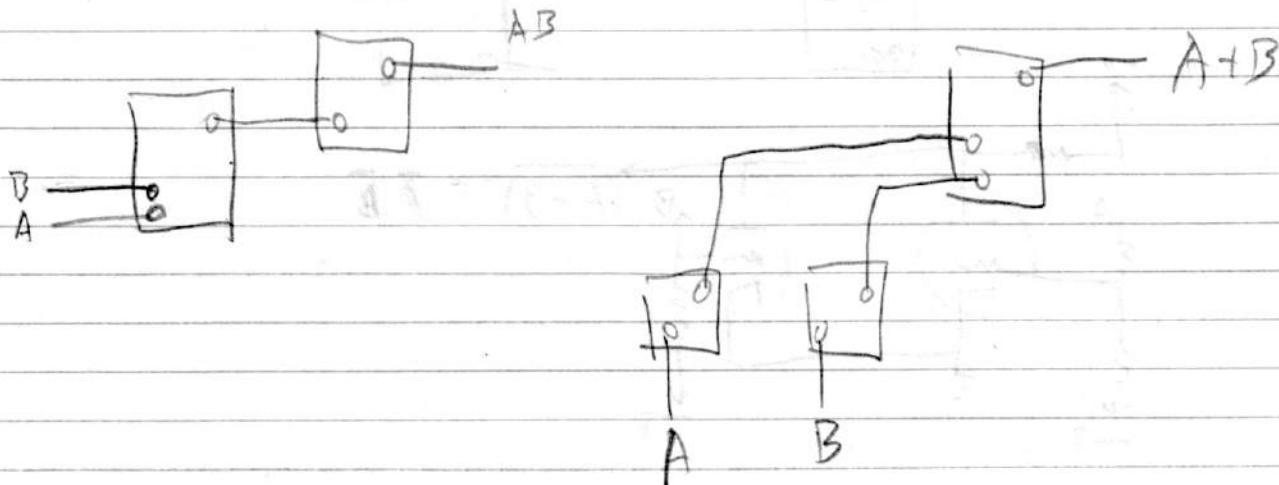
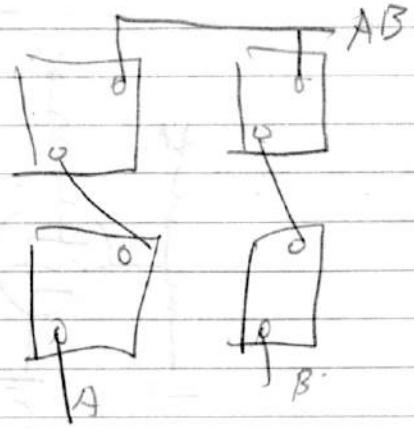
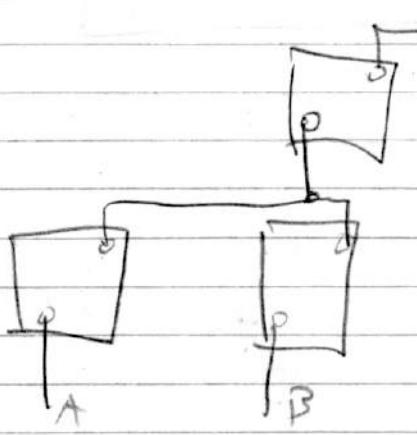
inverter



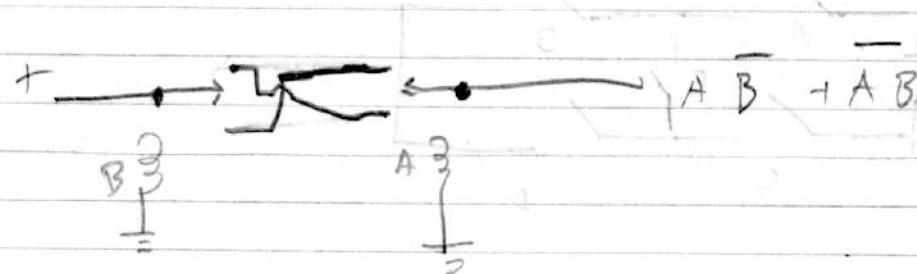
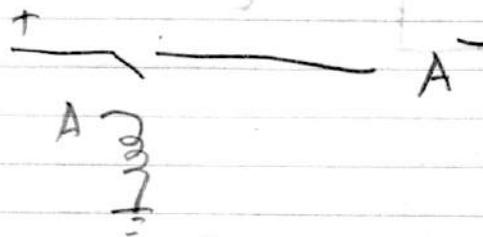
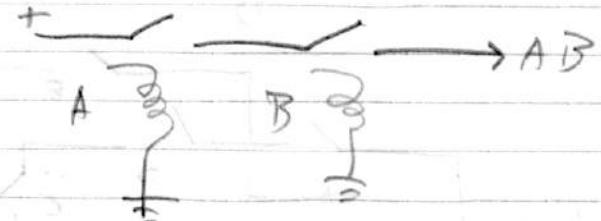
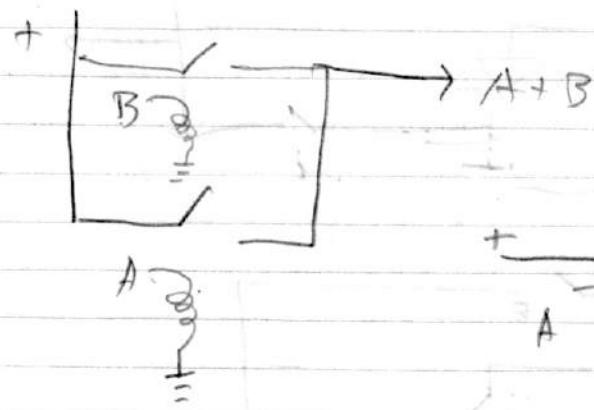
$$(A+B) = \bar{A}\bar{B}$$



As an exercise make up and
and switches using multi-grid
tubes only



Relays



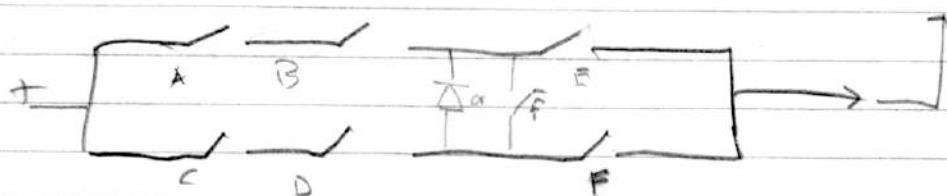
Relay feedback

Relays are useful

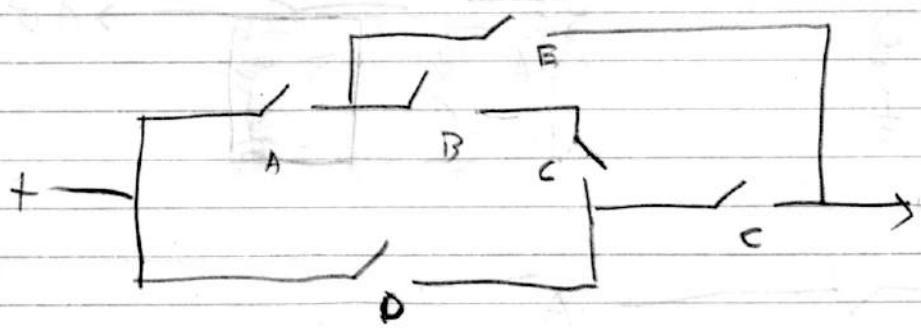
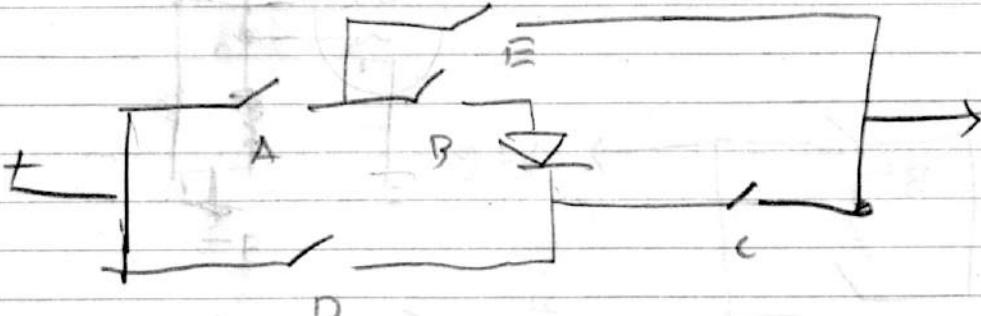
but rather complicated
yielding uncomplicated results

Latching etc...

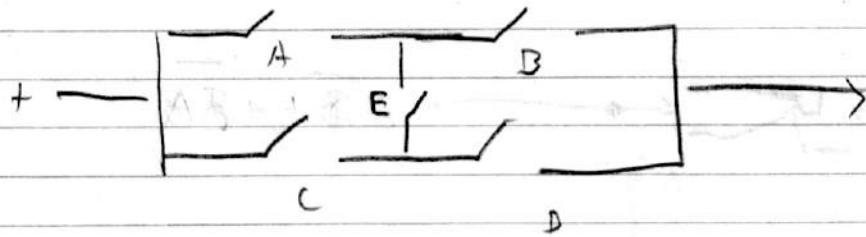
$(AB + CD)E + (CD)F$



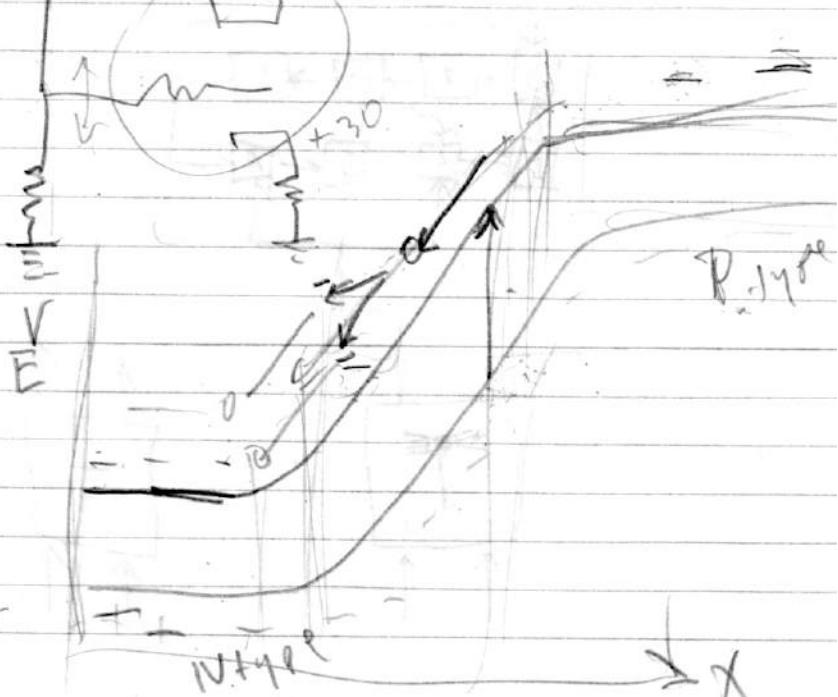
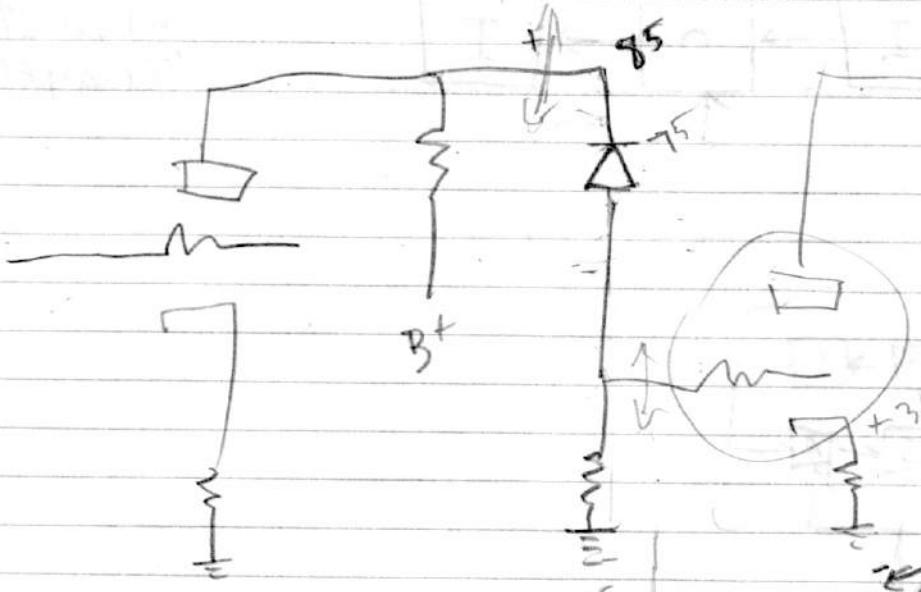
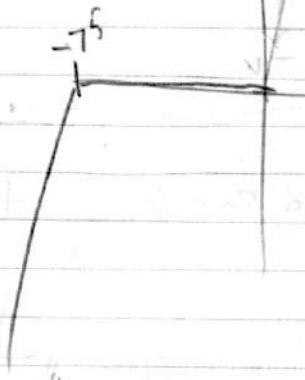
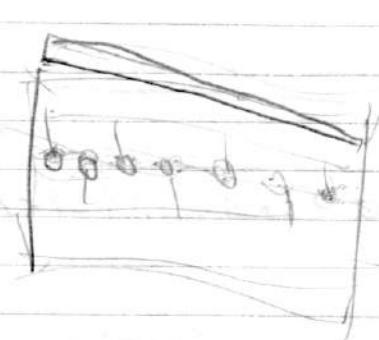
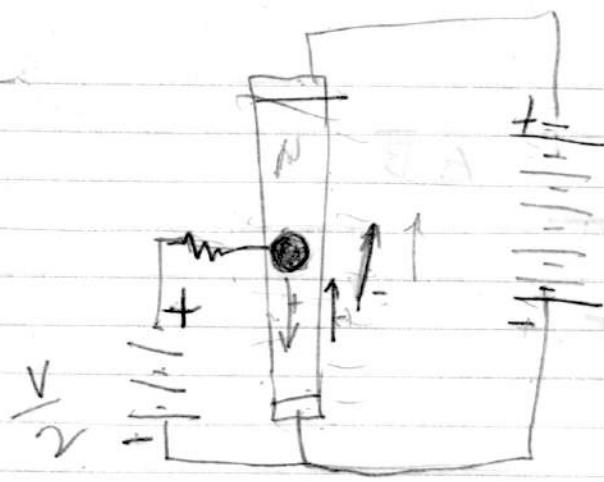
$$(AB+D)C + AE$$



$$AB + CD + AE + CE$$

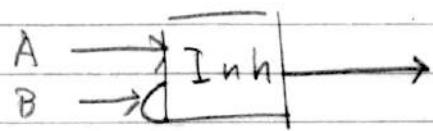


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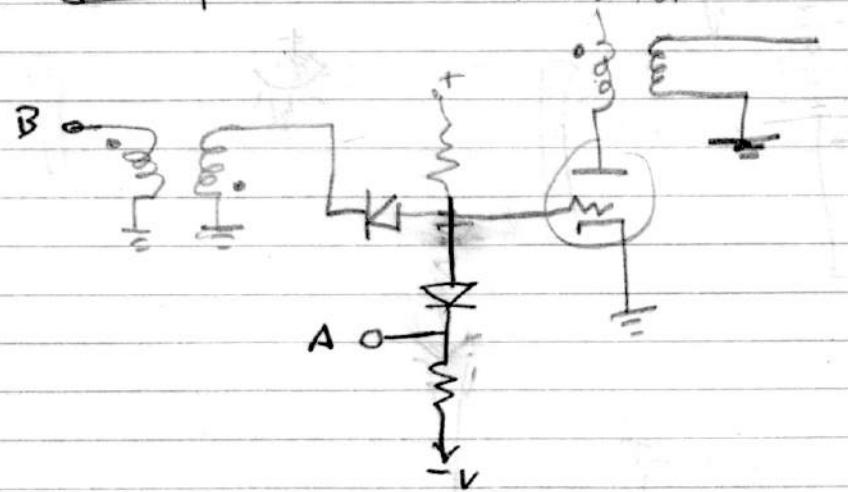
N-type

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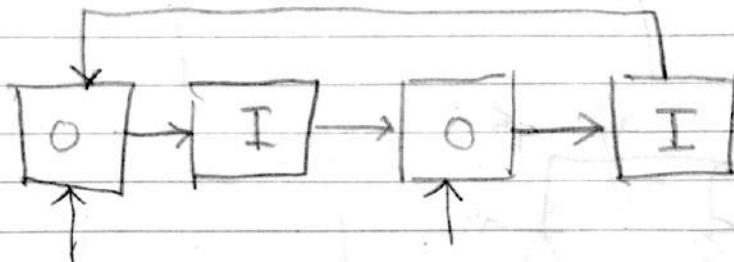


Inhibitor

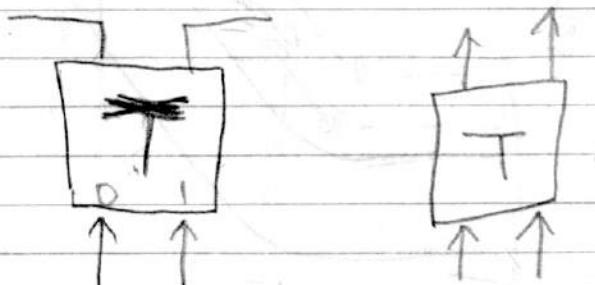
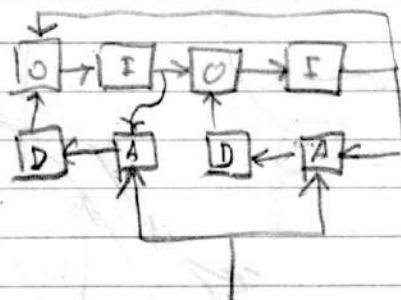
$$A \bar{B}$$

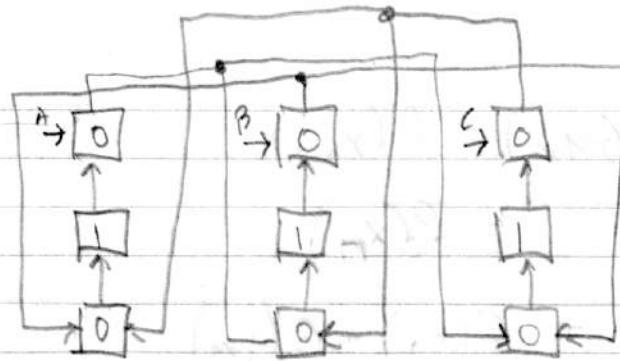


Use of feedback to obtain storage



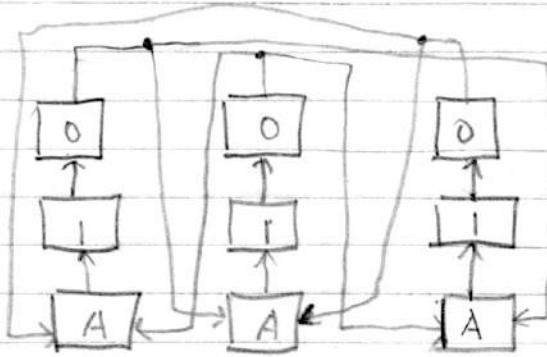
Simple
flip-flop
(Trigger)



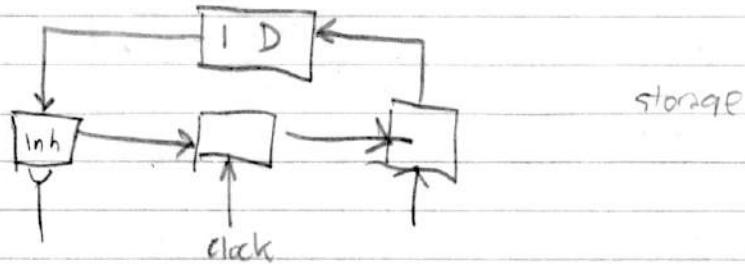


Tri-state Trigger

Develop Tri-state state circuit using
3 inverters, 3 or switches, and 3 & switches



Tri-state trigger



storage

$$f(x_n) \overline{g(y_m)} + \overline{f(x_n)} g(y_m) = f(x_n)$$

$$f(x_n) + \overline{f(x_n)} g(y_m) = f(x_n)$$

$$f(x_n) + \overline{f(x_n)} g(y_m) = f(x_n) + g(y_m)$$

Simplify

$$\begin{aligned} \bar{A}\bar{B} + \bar{A}B &= \bar{A} \\ \bar{A}\bar{B} + A\bar{B} &= \text{can't be reduced} \\ A\bar{B} + AB &= A \end{aligned}$$

$$\begin{aligned} \bar{A}\bar{B} + A\bar{B} + AB &= \bar{B} + AB = A + \bar{B} \\ \bar{A}B + A\bar{B} + AB &= \bar{A}B + A = A + B \end{aligned}$$

$$\begin{aligned} ABC + (\bar{A}\bar{B} + C)(CD + A) &= (AB + C) + (CD + A) \\ &= A + C \end{aligned}$$

$$\bar{A}B + \bar{A}C + \bar{B}C = \bar{A}\bar{B} + \bar{B}C$$

no yes no

$$AB + A\bar{B}C + \bar{A}\bar{B}$$

test +. see if \bar{B} is superfluous. ~~No~~ Yes

$$ACD + A\bar{B}C + \bar{A}BC + \bar{A}C\bar{D}$$

$$ACDB \quad ACD\bar{B} \quad A\bar{B}C\bar{D} \quad \bar{A}BCD \quad \bar{A}BC\bar{D} \quad A\bar{B}C\bar{D}$$

ACD x x

BCD x x

A \bar{B} C x x $\bar{B}C\bar{D}$ x xA \bar{B} C \bar{D} x x $\bar{A}C\bar{D}$ x x

$$\begin{aligned} ACD + \bar{B}C\bar{D} + \bar{A}BC \\ BCD + A\bar{B}C + \bar{A}C\bar{D} \end{aligned}$$

$$ABC + A\bar{B}D + \bar{B}\bar{D}$$

$$ABCD \quad ABC\bar{D} \quad A\bar{B}CD \quad A\bar{B}\bar{C}D \quad A\bar{B}\bar{C}\bar{D} \quad AB\bar{C}\bar{D} \quad \bar{A}B\bar{C}\bar{D} \quad \bar{A}B\bar{C}D$$

ABC X X

ACD X

A $\bar{B}\bar{D}$

BCD X

A $\bar{B}D$

B $\bar{C}\bar{D}$

A $\bar{B}\bar{D}$

B \bar{D}

A \bar{D}

A D

$$ACD + A\bar{B}D + \bar{B}\bar{D}$$

$$ABC + A\bar{B}D + B\bar{D}$$

Problems

Simplify $AB\bar{C} + A\bar{C}D + \bar{A}C + BCD$

$$\bar{A}CD + \bar{A}BC + BCD + \bar{B}D$$

$$ABC + \bar{A}BC + \bar{B}CD + BD$$

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$$\bar{A}\bar{B}\bar{C} + A\bar{C}D + \bar{A}C + B\bar{C}D$$

	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$
$\bar{A}\bar{B}\bar{C}$		x	x					
ABD	x				x			
$\bar{B}\bar{C}D$	x						x	
$\bar{A}\bar{C}D$		x		x				
BED								
$\bar{A}\bar{B}C$						x		x
$\bar{A}\bar{C}D$						x		x
$\bar{A}\bar{B}C$						x	x	x
$\bar{A}\bar{C}D$						x	x	x
$\bar{A}C$					x	x	x	x

$$\boxed{\bar{A}C + A\bar{B}\bar{C} + ABD}$$

$$\bar{A}CD + \bar{A}\bar{B}C + B\bar{C}\bar{D} + \bar{B}\bar{D}$$

	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}\bar{C}\bar{D}$
$\bar{A}\bar{C}D$	x	x						
$\bar{A}BC$	x					x		
BED		x					x	
$\bar{A}\bar{B}D$		x						x
$\bar{A}\bar{B}C$		x	x					
$\bar{A}CD$			x			x		
$\bar{B}\bar{C}D$				x	x			
$\bar{A}\bar{B}D$						x		x
BED						x	x	x
$\bar{B}\bar{D}$								x

$$\bar{B}D + B\bar{C}\bar{D} + \left\{ \begin{array}{l} \bar{A}CD \\ \bar{A}BC \end{array} \right\} + \left\{ \begin{array}{l} \bar{A}\bar{B}C \\ AC\bar{D} \end{array} \right\}$$

$$ABC + \bar{A}BC + \bar{B}CD + BD$$

$$ABCD \quad ABC\bar{D} \quad \bar{A}BCD \quad \bar{A}BC\bar{D} \quad A\bar{B}CD \quad \bar{A}\bar{B}CD \quad A\bar{B}\bar{C}D \quad \bar{A}\bar{B}\bar{C}D$$

ABC	X	X					
B \bar{C} D	X		X				
A \bar{C} D	X			X			
B $\bar{C}\bar{D}$		X		X			
$\bar{A}BC$		X		X			
$\bar{B}D$		X					X
$\bar{A}CD$		X			X		
$\bar{B}CD$	X					X	
$\bar{B}D$							X
BD	X						
CD	X		X		X		
BC	X	X	X	X			

$$BD + CD$$

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$$AC+BC+AD+BD = \bar{A}B + \bar{A}C + \bar{A}D + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{D} + \bar{B}C + \bar{B}D + \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{C}D + \bar{C}\bar{D} + \bar{C}D$$

$$AC+BC+AD+BD = AA + BB + CC + DD + AB + CD$$

= —

$$AC+BC+AD+BD = A(C+D) + B(C+D) = (A+B)(C+D)$$

$$\bar{A}\bar{B}CD + ABCD + AC\bar{B}\bar{D} + A\bar{B}C + \bar{A}BCD + \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}D = \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} +$$

$$= ABCD + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + ABCD$$

AB	X	X					
A \bar{C} D	X			X			
B \bar{C} D		X			X		
$\bar{A}B$		X		O			
$\bar{A}\bar{B}C$			X	O			
$\bar{A}\bar{B}D$			X	X			
$\bar{A}\bar{B}\bar{C}$				X			X
$\bar{A}\bar{B}\bar{D}$					X	X	
$\bar{C}\bar{D}$	O			V	O	X	
$\bar{B}\bar{C}\bar{D}$				V	O	X	
$\bar{A}\bar{C}\bar{D}$					X		X
CD	X	X					

$$\overline{AB} + \overline{CD} = (\overline{AB})(\overline{CD}) \\ = (A+B)(C+D)$$

$$\overline{AB}\overline{CD} + \overline{ABC}\overline{D}$$

(a) $\overline{AC}\overline{D}$
 (b) $\overline{BC}\overline{D}$

} never simplified

$$(ABC\overline{D} + A\overline{B}C\overline{D}) \\ (ABC\overline{D} + A\overline{B}C\overline{D})$$

$$\overline{AB}\overline{D} + A\overline{B}C$$

$$\overline{AB}\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}\overline{D}$$

$\overline{AB}\overline{D}$	x						
\overline{ABC}		x					
$\overline{AC}\overline{D}$			x				
$\overline{BC}\overline{D}$				x		x	x

$$\overline{AB}\overline{D} + A\overline{B}C$$

	B	\bar{B}	
A	X	O	C
\bar{A}	N	N	C
	D	D	\bar{D}

etc

$$ABC\bar{D} = \boxed{\times}$$

$$A\bar{B}C\bar{D} = \boxed{0}$$

$$ABC\bar{C} = \boxed{\times}$$

$$\bar{A}CD = \boxed{M} \quad \boxed{N}$$

$$CD = \boxed{\times}$$

etc.. etc.. etc

	B	\bar{B}	
A	X	X	C
\bar{A}	X	X	C
	D	D	\bar{D}

Simplify

$$ABC + ACD + \bar{A}\bar{C} + \bar{B}\bar{C}D$$

$$\bar{A}\bar{C} + ABC + \bar{A}\bar{B}D$$

	B	\bar{B}	
A	X	X	C
\bar{A}	X	X	C
	D	D	\bar{D}

$$ACD + \bar{A}\bar{B}C + \bar{B}\bar{C}\bar{D} + \bar{B}D$$

$$\bar{B}D + \bar{A}C\bar{D} + ABC$$

	B	\bar{B}	
A	?	*	?
	?		C
	X		
D	D	D	

$$\overline{ABC\bar{D}} + A\bar{B}CD,$$

$\left. \begin{array}{l} ACD \\ BC\bar{D} \end{array} \right\}$ don't care

$$A\bar{B}C + \bar{A}\bar{B}\bar{D}$$

Work

	B			
A	0	*		
	X	0	1	C
				D

$$ABCD + \bar{A}BC\bar{D}$$

only combinations applied will be
 $\left. \begin{array}{l} ABC\bar{D} \\ \bar{A}BCD \end{array} \right\}$ do care

$$AD + \bar{A}\bar{D}$$

ok also through observation
 BC will always appear : the given
 relation will be $AD + \bar{A}\bar{D}$

S	0	S
X	0	

$$ABCD + \bar{A}BC\bar{D}$$

$$\left(\begin{array}{l} \text{applying only} \\ \text{ABC}\bar{D} \\ \bar{A}BCD \end{array} \right)$$

gives

$$\bar{D} + AC$$



$$AC + AD + BC + BD$$

$$\bar{B}\bar{D} + \bar{A}\bar{B}D + \bar{A}B\bar{D}$$

$$(B+D)(A+B+\bar{D})(A+\bar{B}+D)$$

or

$$\bar{A}\bar{B} + A\bar{B}\bar{D} + \bar{A}B\bar{D}$$

$$(A+B)$$

7/29 11:10 - brief talk by S. Surby

or factored form

$$ABCD + \bar{A}BCD$$

$$\begin{array}{l} \text{applying only } AB\bar{C}D \\ \hline \bar{A}BCD \end{array}$$

$$\bar{C} + \bar{A}D$$

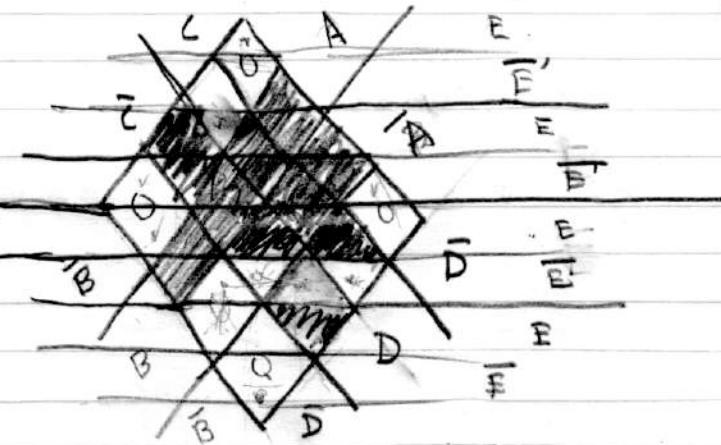
$$C(A+\bar{D})$$

	0		
	x		
x	0		
		1	

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How? ... you have trouble getting
 ~~$\overline{AB} + \overline{AC}D + BC + BDE + \overline{ADE} + \overline{CDE}$~~
 around between more than few variables
 $= (A + BC + DF)(B + AE + ED)$

AB



$$AB + A\overline{C}D + BC + BDE + \overline{ADE} + \overline{CDE}$$

$$= (A + BC + DE)(B + AE + ED)$$

$$(\overline{BD}) + \overline{ABC}\overline{D} + \overline{A}\overline{C}\overline{D}\overline{E}$$

 \overline{ACE} B'

Convert problems of yesterday

(convert) from And to Or to Or to And
by both methods

$$AB\bar{C}\bar{D} + A\bar{B}c + A\bar{C}\bar{D}$$

$\bar{A}B$ is a don't care condition

Find simplest And to or & Or to A forms.

$$A\bar{B}\bar{C} + A\bar{C}D + \bar{A}\bar{C} + BCD$$

$$(A+C)(\bar{A}+\bar{B}+D)(\bar{A}+B+\bar{C})$$

	\bar{B}	\bar{B}	
	X		C
A	X	X	C
	0		C
D	0	0	D

$$\overline{ABCD} \quad ABC\bar{D} \quad A\bar{B}C\bar{D} \quad AB\bar{C}\bar{D} \quad \bar{A}B\bar{C}D \quad \bar{A}\bar{B}\bar{C}D \quad \bar{A}\bar{B}\bar{C}\bar{D} \quad \bar{A}\bar{B}\bar{C}\bar{D}$$

\bar{ABC}	X						
$\bar{BC}\bar{D}$			X				
$\bar{A}\bar{C}\bar{D}$				X			
$\bar{AB}\bar{D}$		X			X		
$\bar{A}\bar{C}D$					X		
\bar{ABC}						X	
$\bar{AB}\bar{C}$							X
\bar{AC}					X	X	X

$$(A+C)(\bar{A}+\bar{B}+D)(\bar{A}+B+\bar{C})$$

ok

$$\bar{A}CD + \bar{A}\bar{B}C + BC\bar{D} + \bar{B}\bar{D}$$

$$(C+D)(\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{D})(\bar{A}+B+D)$$

	\bar{B}	\bar{B}	
0	0	X	D
X	0	X	C
X	X	X	X
0	0	X	C

$$\overline{ABCD} \quad A\bar{B}CD \quad A\bar{B}\bar{C}\bar{D} \quad \bar{A}\bar{B}\bar{C}\bar{D} \quad A\bar{B}CD \quad A\bar{B}\bar{C}D \quad A\bar{B}\bar{C}\bar{D} \quad \bar{A}\bar{B}\bar{C}\bar{D}$$

\bar{ABC}	X	X					
$AB\bar{D}$	X					X	
$\rightarrow B\bar{C}D$	(X)						
$\bar{B}\bar{C}\bar{D}$			X			X	
$\bar{A}\bar{C}\bar{D}$				X			
$A\bar{C}\bar{D}$		X			X		
$B\bar{C}\bar{D}$		X					X
$\rightarrow \bar{C}\bar{D}$		(X)					
$\rightarrow A\bar{B}\bar{D}$		(X)			(X)		
$\rightarrow AB\bar{D}$		(X)			(X)		(X)
$\bar{B}\bar{D}$						(X)	

$$ABC + \bar{A}BC + \bar{B}CD + BD$$

$$(C+D)(B+D)(B+C)$$

	B	\bar{B}	
A	0 X 0 0	0 0 0 0	C
	X X X 0	0 0 0 0	C
	X X X 0	0 0 0 0	C
D	0 X 0 0	0 0 0 0	D
\bar{D}	0 0 0 0	0 0 0 0	D

ABCD	ABC \bar{D}	A $\bar{B}C\bar{D}$	A $\bar{B}\bar{C}D$	A $\bar{B}\bar{C}\bar{D}$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}\bar{C}D$
A $\bar{B}\bar{C}$	X			X				
A $\bar{B}D$	X		X					
A $\bar{C}D$	X	X						
$\bar{B}CD$	X				X			
$\bar{A}BC$					X			X
$\bar{A}\bar{C}D$					X	X		
A $\bar{B}D$					X		X	
$\bar{B}C$	X			X	X			X
$\bar{B}D$	X		X		X			X
$\bar{C}D$	X	X			X	X		

$$(C+D)(B+D)(B+C)$$

OK

$$ABC\bar{D} + A\bar{B}C + A\bar{C}D$$

$\bar{A}\bar{B}$ - don't care

X		X
X	X	X
D	D	
D	O	

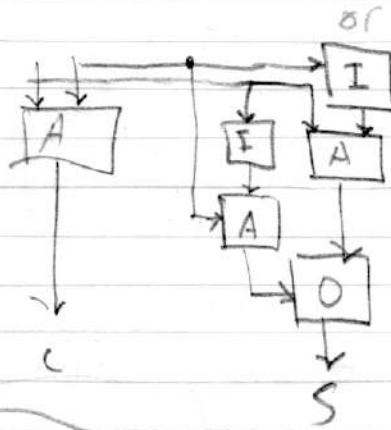
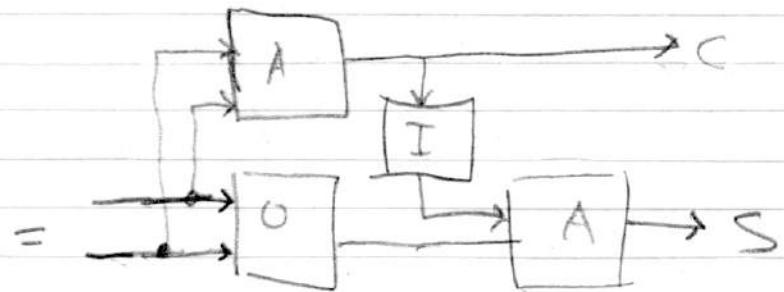
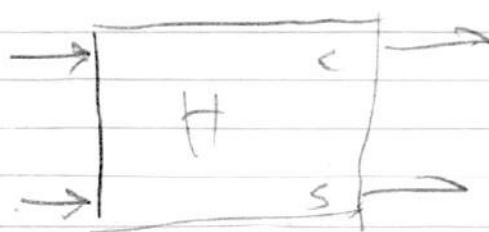
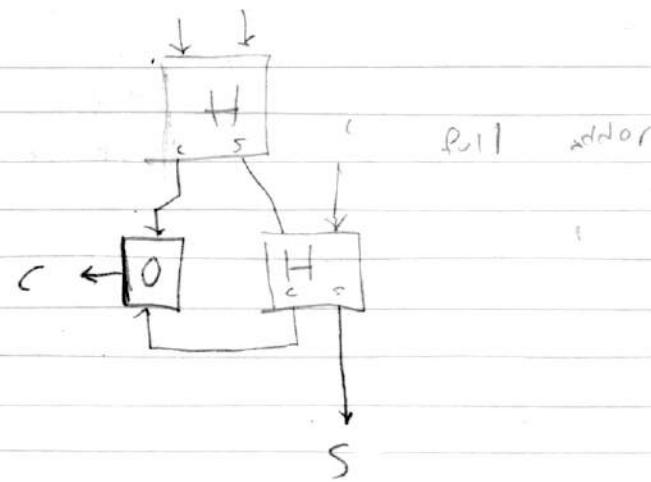
$$A(\bar{B} + \bar{D})(C + \bar{D})$$

0 → A

$$BD + AD + A\bar{B}C$$

A → G

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Half Adder

$$S = (X+Y)\bar{XY} = X\bar{Y} + Y\bar{X}$$

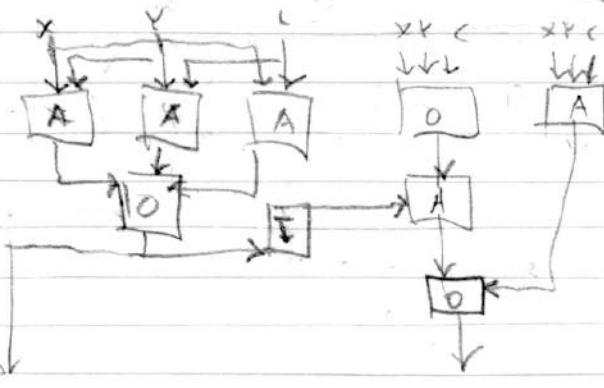
$$C = XY$$

Full Adder

$$\text{Sum} = \bar{C}x\bar{y} + \bar{C}x\bar{y} + Cx\bar{y} + Cxy$$

$$\text{Carry} = \bar{C}xy + C\bar{x}y + Cx\bar{y} + Cxy$$

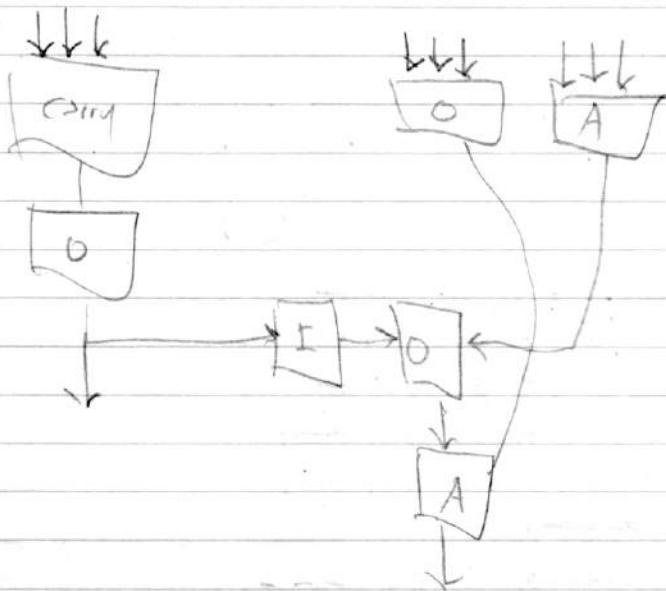
Full Adder



Full Adder

$$\text{Carry} = XY + XC + YC$$

$$\text{Sum} = (X+Y+C)(\cancel{XY+XC+YC}) + XYC$$



$$S_{(1)} = XY + CY + CX$$

$$\text{sum} = (X+Y+C)[\overline{(XY+XC+YC)} + XC]$$

Show $\bar{C} \bar{X} \bar{Y} + \bar{C} \bar{X} Y + C \bar{X} \bar{Y} + C X Y$

is equivalent to $(X+Y+C)[\overline{(XY+XC+YC)} + XC]$

by Boolean Algebra

Show by Boolean Algebra that you get a full adder with two half adders & an OR switch

$$\text{Show } \bar{x}\bar{y} + \bar{c}\bar{x}y + c\bar{x}\bar{y} + xy = (x+y+c) \left[(\bar{y}+\bar{x}+c) + xyc \right]$$

$$= \overline{(x+y+c)} + (\bar{y}+\bar{x}+c) + xyc$$

$$= \overline{\bar{x}\bar{y}\bar{c}} + xyc + \bar{y}\bar{c} + \bar{y}c + \bar{x}\bar{c} + \bar{x}c + \bar{x}\bar{y}c$$

$$= \overline{\bar{x}\bar{y}\bar{c}}$$

$$= (x+y+c) \left[\overline{\bar{x}\bar{y}\bar{c}} + \bar{y}\bar{c} + \bar{y}c + \bar{x}\bar{c} + \bar{x}c + \bar{x}\bar{y}c \right]$$

$$\bar{x}\bar{y}\bar{c} + xyc + \bar{y}\bar{c} + \bar{x}\bar{y}c = \bar{x}\bar{y}c + \bar{x}\bar{c} + \bar{y}\bar{c} + \bar{x}\bar{y}\bar{c}$$

x	x	x	x
x			
c	c	c	c
	y	y	y

$$= (x+y+c)(\bar{x}\bar{y}\bar{c} + \bar{x}\bar{y}c + \bar{x}y\bar{c} + \bar{x}\bar{y}\bar{c} + xyc)$$

$$= x(xyc) + y(\bar{x}yc) + c(\bar{x}\bar{y}c) + xyc$$

$$= x\bar{y}\bar{c} + \bar{x}yc + \bar{x}\bar{y}c + xyc$$

$$\text{SUM} \quad (\bar{x}\bar{y} + \bar{y}\bar{x})\bar{c} + \bar{c}(\bar{x}\bar{y} + \bar{y}\bar{x}) \quad \left| \begin{array}{l} \text{show} \\ = \bar{c}\bar{x}\bar{y} + \bar{c}\bar{x}\bar{y} + \bar{c}\bar{x}\bar{y} + \bar{c}\bar{x}\bar{y} \end{array} \right.$$

$$\bar{c}\bar{x}\bar{y} + \bar{x}\bar{y}\bar{c} + c(x\bar{y} + \bar{x}\bar{y}) =$$

$$\bar{c}\bar{x}\bar{y} + \bar{x}\bar{y}\bar{c} + c\bar{x}\bar{y} + c\bar{x}\bar{y} = \text{QED.}$$

CARRY

$$x\bar{y} + c(x\bar{y} + \bar{x}\bar{y}) = \bar{c}x\bar{y} + c\bar{x}\bar{y} + c\bar{x}\bar{y} + c\bar{x}\bar{y}$$

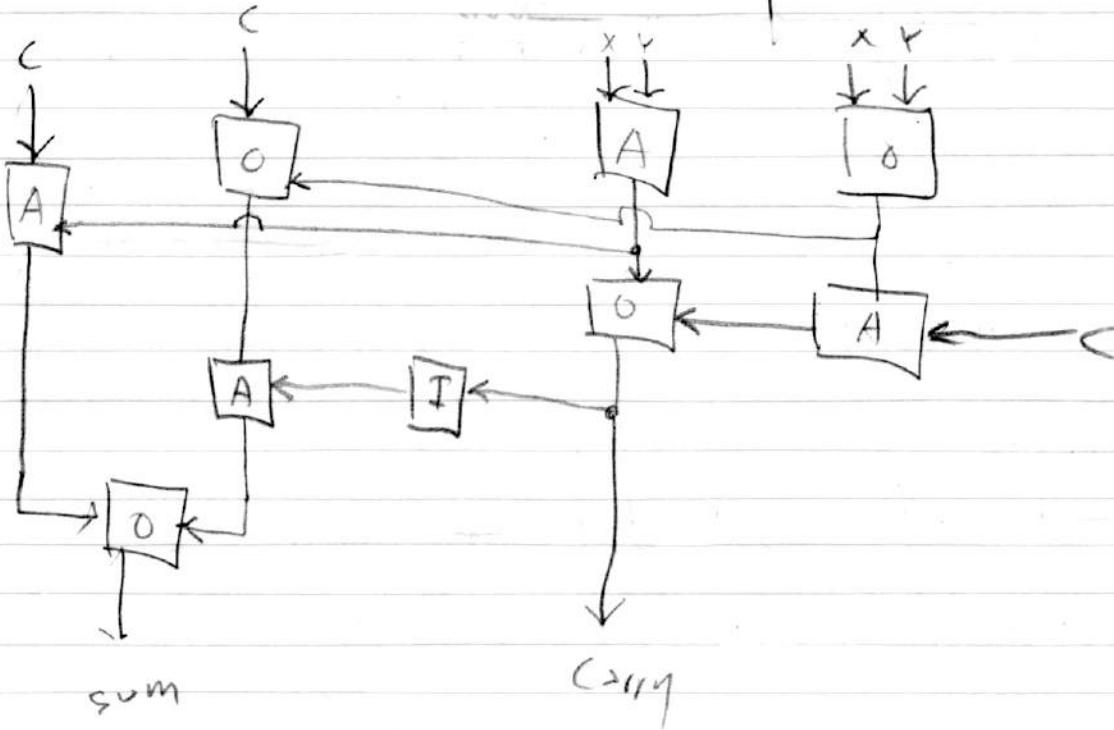
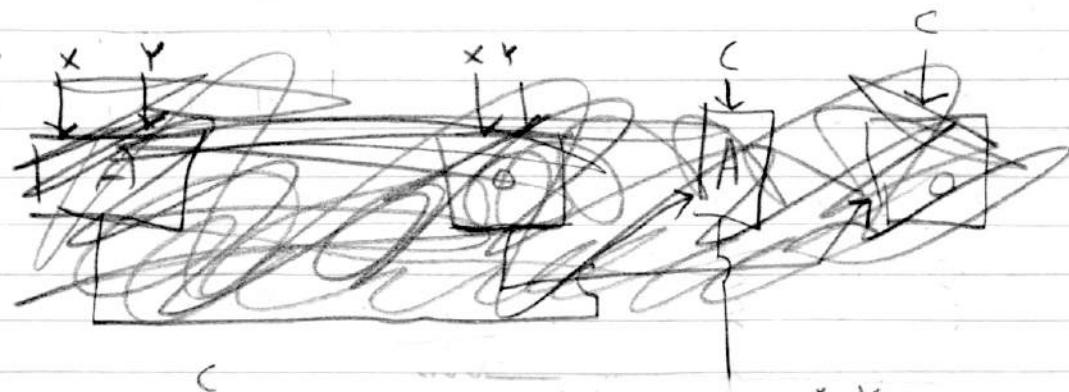
$$xy\bar{c} + x\bar{y}\bar{c} + c\bar{x}\bar{y} + c\bar{x}\bar{y} = \text{QED}$$

$$\text{SUM} = (x+y+c) (\overline{xy+yc+xc}) + xyC$$

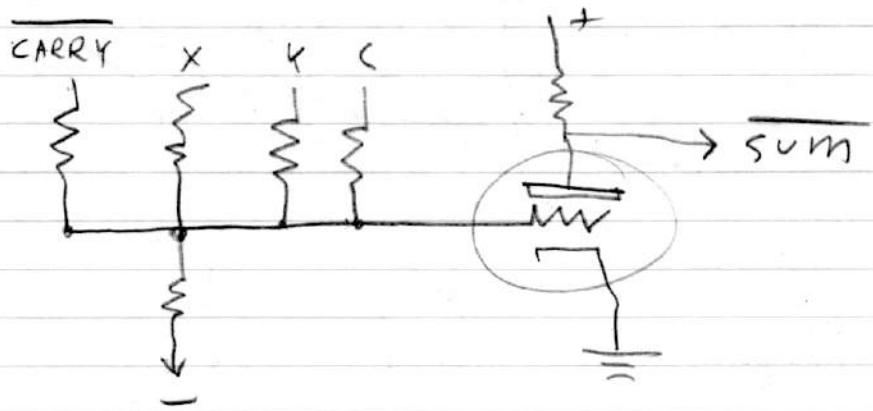
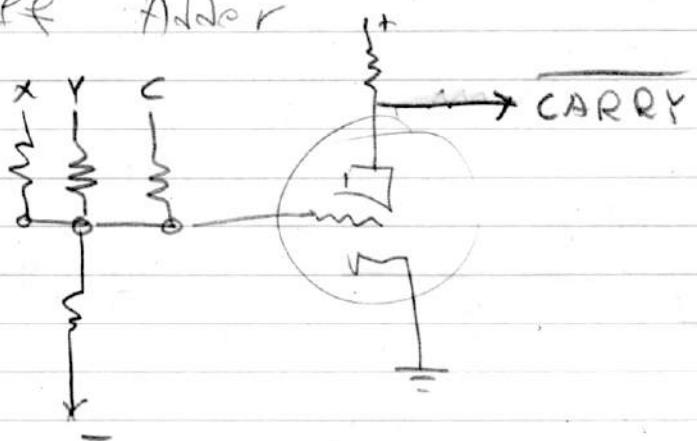
$$\text{CAR} = xy + xc + yc$$

$$\text{CAR} = xy + (x+y)c$$

$$\text{SUM} = [(x+y)+c] (\overline{xy+(x+y)c}) + xyC$$

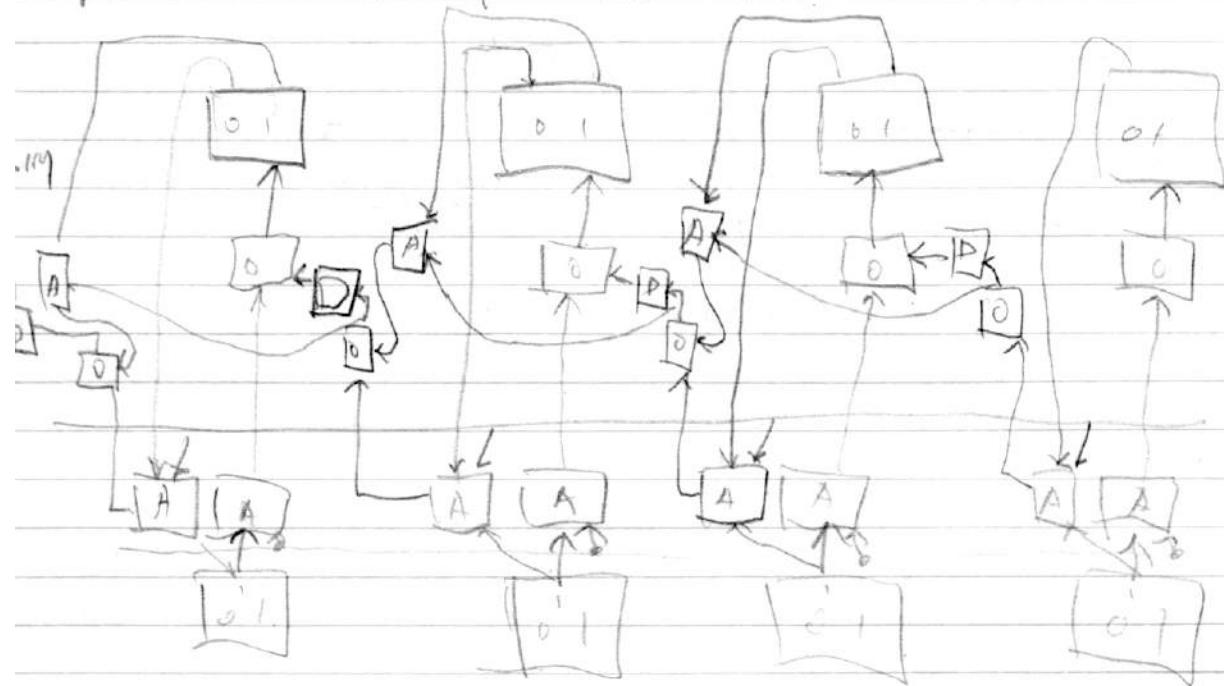


Kirchoff Adder



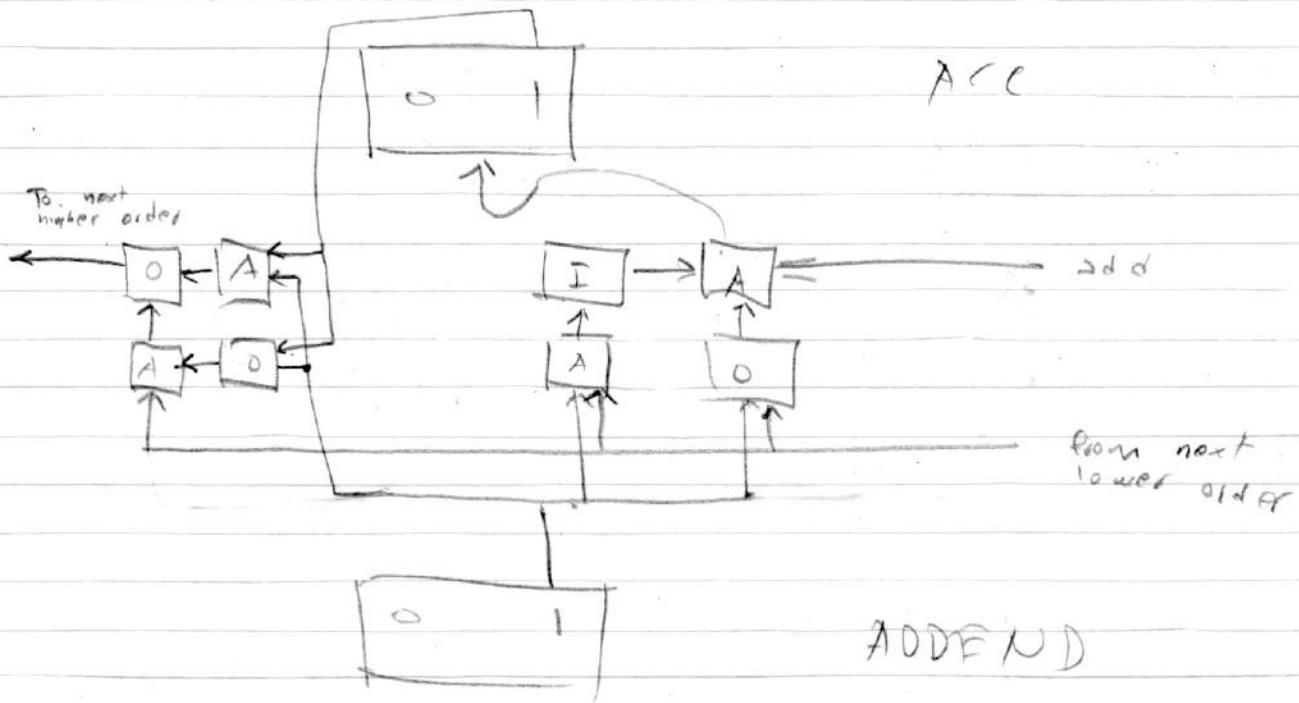
Addition through an accumulator.....

parallel.....

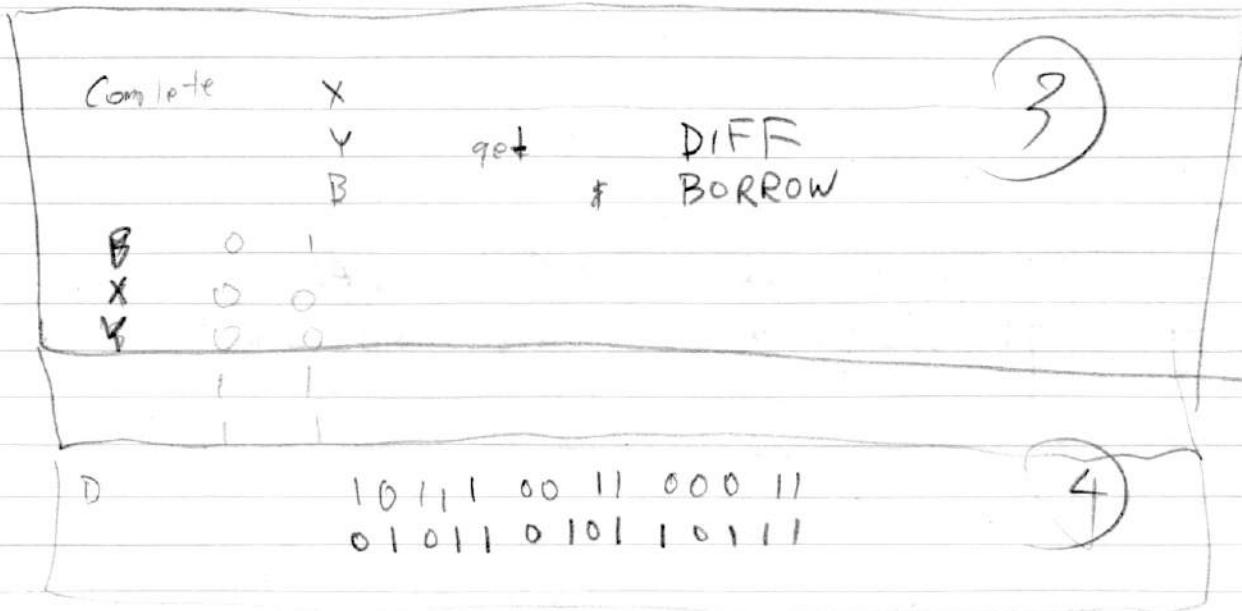


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701 - 36 full adders - carry propagation issue



Subtraction & compliment addition

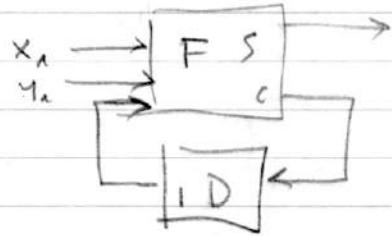


subtraction by 1's compliment addition

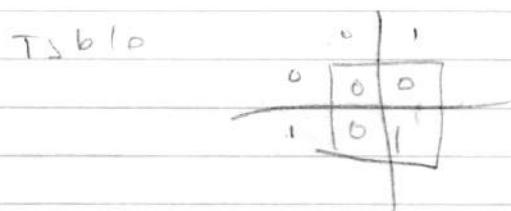
$$1's \text{ comp}_N = 2^N - N - 1$$

Positive numbers are used to add in complement form often. This yields a positive zero.

Serial adder

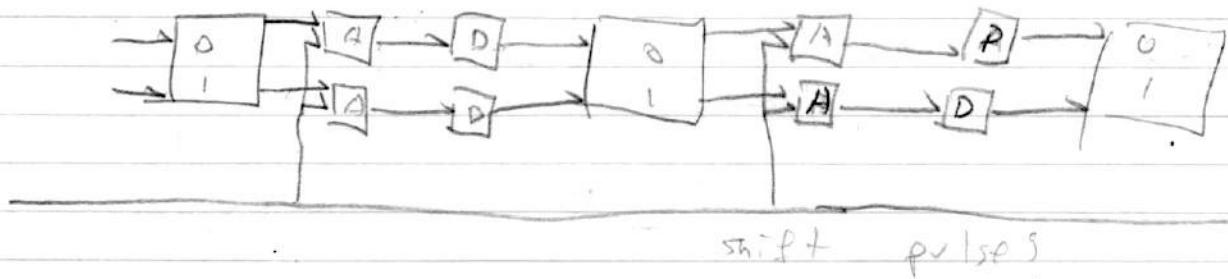


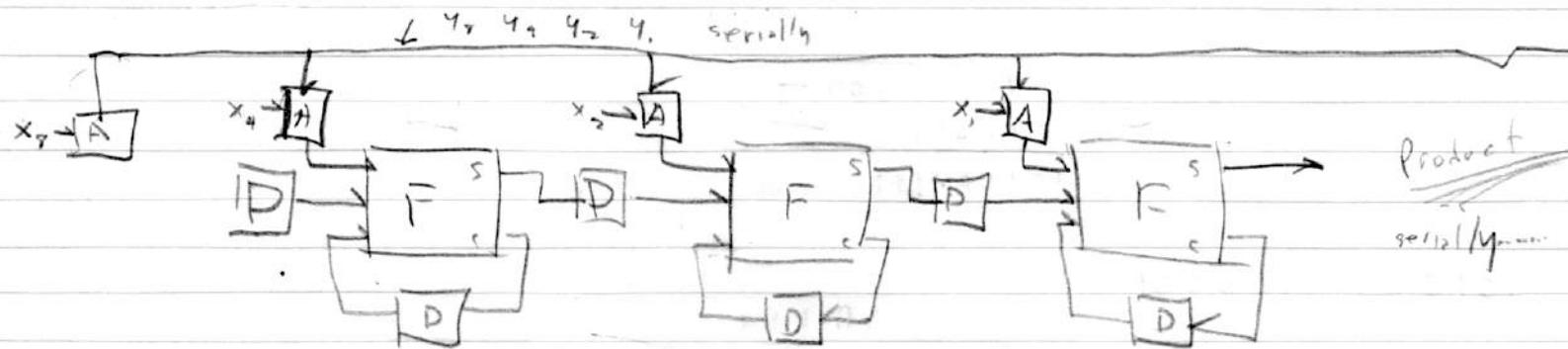
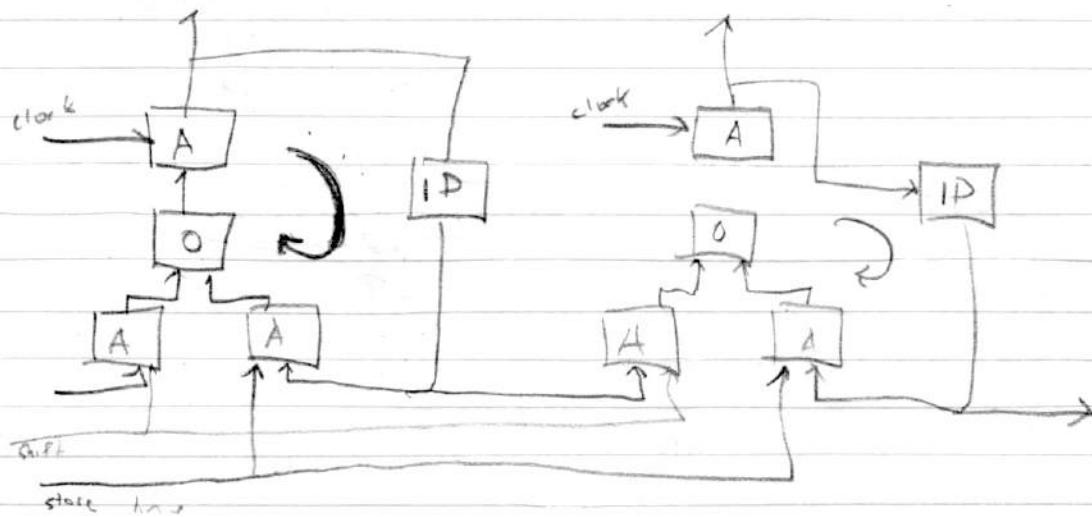
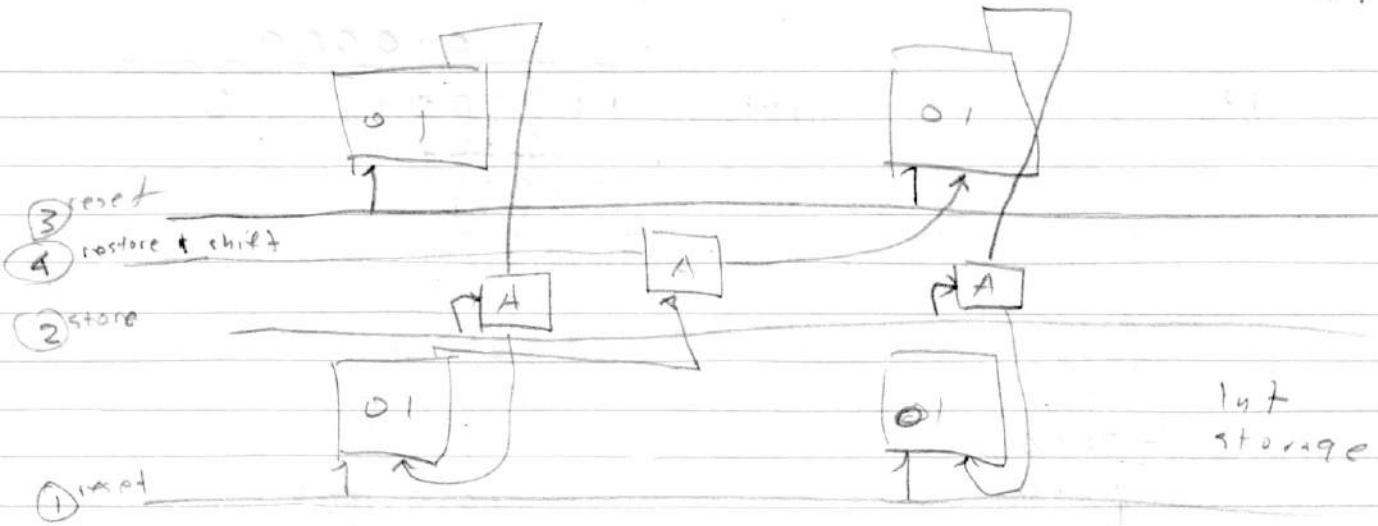
Binary Multiplication



8 | 2

shift Register





Hi-speed binary multiplier ↑

Division ...

$$\begin{array}{r}
 0100001 \\
 \overline{)01111001110} \\
 -1101 \\
 \hline
 101 \\
 -1001 \\
 \hline
 1101 \\
 -1100 \\
 \hline
 1101 \\
 -10001 \\
 \hline
 1101 \\
 -1101 \\
 \hline
 000000000011 \\
 \end{array}$$

$$\begin{array}{r}
 1110011 \\
 \times 1011011 \\
 \hline
 1110110111001110
 \end{array}$$

use 2 ↑ methods

$$\begin{array}{r}
 1110011 \\
 1110011 \\
 \hline
 10111001 \\
 101011001 \\
 \hline
 110000100001 \\
 1110011 \\
 \hline
 10100011100001
 \end{array}
 \quad
 \begin{array}{r}
 1110011 \\
 1110011 \\
 \hline
 101011001 \\
 101011001 \\
 \hline
 110000100001 \\
 1110011 \\
 \hline
 10100011100001
 \end{array}$$

10100011100001

$$\begin{array}{r}
 00111 \\
 \overline{)01111001110} \\
 -111011 \\
 \hline
 -0111111 \\
 \hline
 111011 \\
 -1001 \\
 \hline
 111011 \\
 +1100101 \\
 \hline
 111011 \\
 +1010100 \\
 \hline
 111011 \\
 \end{array}$$

TEST #2

July 30, 1954

TRUE OR FALSE

F

1. ✓ A modified binary pulse counter such as used in the 604 provides a carry pulse when it changes from one to zero.

F

2. ✓ A Factor or General Storage position is reset to zero on a Read Out operation.

T

3. ✓ When the + on CS4 voltage is available to the column shift unit information on the storage exit channel (not the counter exit channel) is shifted three positions with respect to the entry channels.

T

4. ✓ A number in the Counter or General Storage always punches out in true form.

T

5. ✓ To cause punching out from the counter, a counter Read Out on the punch control panel is plugged.

T

6. ✓ On a storage to storage transfer of information "A" pulses are used for Read Out and "B" pulses are used for Read In.

F

7. ✓ Nine pulses are fed into a storage position on an electronic Read-Out operation.

T

8. ✓ The Punch Read-Out pulses are produced as a result of a trigger which is controlled by cams (CB's) in the punch.

F

9. ✓ A positive number in the counter is always in true form.

T

10. ✓ Any number in a storage unit is always in true form.