Report ~~48-11~~
R.L. 184

October 29, 1942

Radiation Laboratory

THEORY OF HIGH FREQUENCY RECTIFICATION

BY SILICON CRYSTALS

[redacted]

Abstract

The excellent performance of British "red dot" crystals is explained as due to the knife edge contact against a polished surface.

High frequency rectification depends critically on the capacity of the rectifying boundary layer of the crystal, C. For high conversion efficiency, the product of this capacity and of the "forward" (bulk) resistance R_b of the crystal must be small. For a knife edge, this product depends primarily on the breadth of the knife edge and very little upon its length. The contact can therefore have a rather large area which prevents burn-out. For a wavelength of 10 cm. the computations show that the breadth of the knife edge should be less than about 10^{-3} cm.

For a point contact the radius must be less than 1.5×10^{-3} cm. and the resulting small area is conducive to burn-out. The effect of "tapping" is probably to reduce the area of contact.

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THEORY OF HIGH FREQUENCY RECTIFICATION BY SILICON CRYSTALS

Introduction

This paper deals only with those phenomena in high frequency rectification which are due to the combined effect of conductivity and capacity. It does not deal with the solid state theory of rectifying contacts which is treated in a separate report (43-12). A knowledge of that report is not necessary for the understanding of the present paper.

In the report mentioned it is shown that a consistent theory of the contact can be obtained by assuming that the surface of the crystal has the same chemical composition as the interior, i.e., without the assumption of an artificial blocking layer. It is necessary to assume a high dielectric constant such as is usually found for non-metallic elements. In all numerical calculations we shall assume a dielectric constant $\epsilon = 10$. Using the measured number of conduction electrons for "average" silicon, the thickness of the boundary layer then comes out to be of the order of 10^{-6} cm. The motion of electrons through the boundary layer is essentially like that in a diode, since the mean free path of electrons is about 5×10^{-6} cm., i.e., several times the thickness of the layer.

The characteristics of the contact for D.C. are the usual ones: in the forward direction the current increases exponentially with the applied voltage up to a certain voltage φ_o; if the voltage is greater than φ_c, any amount of current can pass through the contact (remember, however, the bulk resistance discussed below!). In the backward direction, the current reaches a small saturation value for a back voltage of about .1 volt; if the back voltage is increased to about one volt or more, breakdown occurs due to the Schottky effect (image force.)

The actual resistance in the forward direction for voltages greater than φ_c is determined by the bulk resistance of the crystal. If the contact area is a circle of radius a, the resistance is, according to a well-known formula of potential theory,

$$R = \frac{1}{4\sigma a} \qquad (1)$$

where σ is the conductivity. According to experiments of Dr. Becker* of the Bell Telephone Laboratories, the area of contact derived from Eq. (1) agrees well with that measured under a microscope. The resistance R can easily be obtained experimentally from the slope of the straight part of the D.C. current voltage characteristics. Values of R in the neighborhood of 20-50 ohms are common. For the back resistance, very much higher values of several thousand ohms are usually obtained for good rectifiers.

The boundary layer acts as a capacity in addition to having a conductance. As the potential across the boundary layer is increased, more donators in the surface layer must be charged up so as to increase the thickness of the layer. If the potential decreases, the donators will return to a neutral condition. It can easily be shown (cf. Report 43-12) that the effective capacity of the boundary layer is

$$C = \frac{A\epsilon}{4\pi d} \qquad (2)$$

where A is the area of the contact and d the thickness of the boundary layer. This thickness depends slightly on the applied voltage, being given by

$$d = \sqrt{\frac{\epsilon(\varphi_0 - \varphi_a)}{2\pi N_d e}} \qquad (3)$$

where N_d is the number of donators per cm^3, e the electronic charge, φ_0 the difference of the work functions of metal and semiconductor, and φ_a the applied voltage in the forward direction. This dependence on φ_a, however, is not very important. Measurements of the capacity** using intermediate frequency give values of the order of 1 μμf. With $d = 10^{-6}$ and $\epsilon = 10$ this gives an area A of about 10^{-6} cm^2.

*I am obliged to Dr. Becker for [illegible]

**I am indebted to Dr. Beringer of [illegible] Radiation Laboratory for telling me the results of his capacity me[illegible] which is of the same order as that deduced from Eq. (1) or from geometric measurements of the size of the point under the microscope.

Capacity and R.F. Rectification

If the contact is exposed to high frequency fields, the capacity will shunt out the resistance of the contact in the backward direction. This is easily seen because a capacity of 1 μμf corresponds to a resistance of only 50 ohms at $\lambda = 10$ cm. Therefore, a high back resistance is of no use for the high frequency rectifying action of the contact, but the capacity of the contact is the significant quantity.

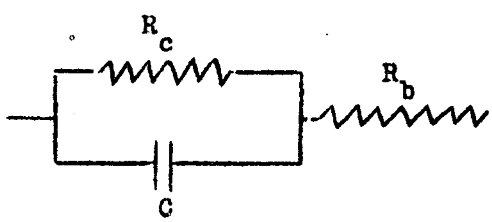


Fig. 1

The equivalent circuit of the contact is given in Figure 1. The contact itself is equivalent to a resistance R_c in parallel with a capacity C. The latter quantity is slightly dependent upon the applied voltage (Eq. 3), the former quantity depends very strongly on φ_a. In series with this element is the bulk resistance R_b which is given by Eq. (1) and is independent of the applied voltage. For reasonably large forward voltage R_c is very small so that the effective impedance of the contact proper is 0 and the total impedance is equal to R_b. For any backward voltage and also for small forward voltages the resistance R_c is very large so that the effective impedance of the contact proper is $\frac{1}{i C \omega}$.

Then the total impedance is

$$Z = R_b + \frac{1}{i C \omega} \qquad (4)$$

The ratio of back to forward impedance which is significant for the rectification is

$$\frac{Z}{R_b} = 1 + \frac{1}{iC\,R_b\omega} \qquad (5)$$

It is thus seen that the significant quantity for high frequency rectification is CR_b rather than the back-to-front ratio for D.C., $R_c(\text{back})/R_b$.

This explains the erratic behavior of the relation between D.C. and R.F. rectification. It shows that nothing is gained by working for extremely high back-to-front ratios for D.C. However, as we shall see in the following, the back resistance for D.C. has some relation to the capacity of the contact and for this reason gives some indication of the R.F. characteristic to be expected.

In order to find the necessary requirements for a good rectifying contact, we use Eq. (1) and (2) and obtain

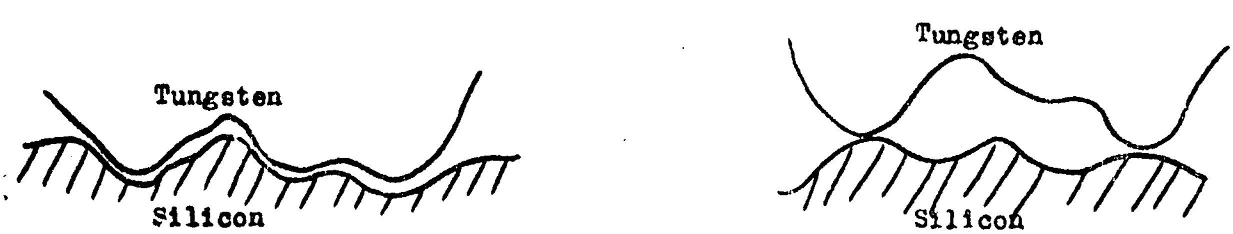
$$R_b C\omega = \frac{2\pi}{30} \times \frac{A}{4\pi\, d\lambda} \times \frac{1}{4\sigma a} \qquad (6)$$

In this formula, λ is the wave length of the R.F. and 30 represents the "velocity of light in ohms." In order to get a good rectification, Expression (6) must be as small as possible, preferably smaller than 1. It is seen immediately that this condition becomes increasingly more difficult to satisfy as the wave length decreases. If we assume a circular contact of radius a, then the right-hand side of Eq. (6) is proportional to a. If we take $\lambda = 10$ cm., $d = 10^{-6}$ cm., $\epsilon = 10$, $\sigma = 20\ \text{ohm}^{-1}\ \text{cm.}^{-1}$, and require Expression (6) to be less than 1, we obtain the condition

$$a < \frac{240}{\pi}\, d\lambda \frac{\sigma}{\epsilon} = 1.5 \times 10^{-3}\ \text{cm.} \qquad (7)$$

This shows that good rectification can only be obtained with small point contacts. The smaller the area of the contact, the better should be the conversion efficiency of the contact at high frequency. It is, of course, true that (7) is only a necessary and not a sufficient condition for good rectification. It is also necessary to have a local oscillator of sufficient amplitude in order to be able to make full use of the rectifying properties of the contact and, in turn, the local oscillator power may be limited by the noise it generates in the crystal. However, given the power of the local oscillator, the size of the contact has to be made small. The contacts actually used fulfill condition (7) by a considerable margin. A capacity of 1 cm. which corresponds to the average of the rectifiers measured by Dr. Beringer will give $a = 6 \cdot 10^{-4}$ cm. so that Expression (6) has the value 0.4. Lawson, of the University of Pennsylvania, has studied contacts with a varying from about $3 \cdot 10^{-4}$ to 10^{-3} cm.

The requirement of small point contact explains the function of tapping* in producing a good rectifier. As the cat's whisker is brought into contact with the crystal surface for the first time, the relatively soft tungsten is caused to flow plastically and to assume the shape of the silicon surface. Then the tapping will move the cat's whisker to a different area on the silicon which will have different irregulatities from the area first touched. Therefore, the cat's whisker will now make contact only in a few points rather than over the entire flattened surface (Fig. 2). This will obviously improve the rectifying characteristics.



Before Tapping   After Tapping

Fig. 2

*The following picture of the phenomena involved in tapping was suggested to me by Dr. Schiff of the University of Pennsylvania.

A measure of the size of the contact area is given either by the forward resistance or the back resistance for D.C. The forward resistance is inversely proportional to the diameter of contact (Eq. 1). The back resistance is inversely proportional to the area of contact. For this reason, the back resistance is a more sensitive indicator of the size of the contact. Moreover, the forward resistance will be reduced if there is contact at more than one point. A small forward resistance caused in this way will not be detrimental to the rectifying characteristics of the contact because they depend only on the individual size of each contact. The ratio of back-to-front resistance will, however, not depend on the number of contact points but will give a direct indication of the average size of the individual contact. Therefore, the back-to-front ratio will be a better measure of the rectifying characteristic of the contact for R.F. than the forward D.C. r resistance. Therefore, although a high back-to-front ratio is of no direct use for high frequency rectification, it is still indicative of small point contact and therefore will probably indicate a good rectifying contact for high frequency as well as for D.C.

It must be remembered, however, that the back-to-front ratio does not depend on the contact area alone. The back resistance depends very sensitively on the difference of the work functions of metal and semiconductor which may be very sensitive to surface layers of oxygen and the like. Furthermore, it will be influenced by fluctuations in the thickness of the boundary layer and other reasons which cause local concentrations of the electric field and thus enhance the Schottky effect (cf. Report 43-12). The average donator density will also influence the back resistance more strongly than the capacity of the contact.

Burn-out

The small area of contact which is required for good rectification is undesirable from the standpoint of burn-out. The power going through the crystal will be dissipated in a smaller volume if the area of contact is smaller and will therefore heat the boundary layer to a higher temperature. At such a high temperature there will be diffusion of donators in the crystal which will spoil the rectifying characteristics as described in the accompanying report (43-13). Possibly even the tungsten may diffuse into the crystal and thereby cause too high conductivity of the boundary layer.

Dr. Torrey of this Laboratory has calculated that the temperature reached in the boundary layer is given by the formula

$$\theta = \frac{P R_b \sigma}{4.2 \, (k_c + k_M \tan \Psi/2)} \qquad (8)$$

Here P is the power (in watts) going through the crystal, 4.2 is the conversion factor from joules into calories, σ is the electric conductivity of the crystal, k_c is its heat conductivity in the usual units (calories/cm.^{2} per degree/cm.), k_M is the same for the metal and Ψ is the half angle of the cone forming the tip of the cat's whisker. For silicon, k_c is about .1; for tungsten, $k_M \approx 0.4$. The most important feature of Eq. (8) is that, for Si of given conductivity σ, θ is proportional to the forward resistance of the contact, R_b. Therefore, the temperature will be higher for a smaller point which has a high forward resistance. This agrees with the experience that the best rectifying crystals usually are most sensitive to burn-out. It therefore seems difficult to produce crystal rectifiers which are at the same time sensitive and stable against burn-out.

Knife Edge Contacts

However, there is one way out of this dilemma. The quantity which must be made small in order to achieve good rectification is C x R_b. Now C is strictly proportional to the area of the contact owing to the very small thickness of the boundary layer. R_b, however, depends on the shape of the area of contact. If we make contact in a knife edge rather than in a circular point the resistance will be very nearly inversely proportional to the length of the knife edge and almost independent of its breadth. The former fact is seen if we consider a knife edge of given breadth b and different length $a \gg b$. Then for a given applied potential the current must be nearly proportional to a. Moreover, the resistance is dimensionally given by the resistivity divided by some length, so that there is no room for any strong dependence on the breadth of the knife edge. Dr. Torrey has calculated the resistance for a contact whose cross section is an ellipse with semi-axes a and b. For $a \gg b$, he finds

$$R_b = \frac{\ln 4a/b}{2\pi \sigma a} \qquad (9)$$

Therefore, the important quantity for rectificationsbecomes now

$$CR_b \propto AR_b \propto b \ln 4a/b \qquad (10)$$

It is therefore almost independent of the major dimension a and depends essentially only on the smaller dimension b. Therefore, it is possible to obtain a good rectifying contact by using a knife edge whose breadth is small compared with the critical dimension given in Eq. (7), but whose length may be quite large compared with that quantity. Such a contact will have a very small forward resistance R_b according to Eq. (9.. It will therefore have a low temperature for any given transmitted power and will consequently be very stable against burn-out.

It appears that knife edge contacts have been made and investigated by the British General Electric Company, Ltd. They have invariably been found to possess as good rectifying properties as the better American rectifiers, and to be capable of sustaining powers of as much as 5 watts for long times. This excellent performance was never understood up to the present. It appears to be due not so much to the chemical and heat treatment of the crystals as simply to the use of a knife-edge contact. It must be kept in mind, however, that a knife edge can make good contact with the crystal only if the crystal surface is very accurately plane and extremely smooth. Therefore, we believe that the British treatment of the crystal has mainly the effect of making its surface plane.

Crystal Properties Affecting R.F. Rectification

In view of the possibility of getting reliable, sensitive and sturdy crystal rectifiers by means of a knife edge contact it is hardly worth while to discuss the other factors which influence the performance of the crystal. Most of these factors are contained in the formula for the thickness of the boundary layer, Eq. (3).

From this formula it would appear that better rectification would be obtained by having a smaller density of donators, N_d. However, the conductivity σ is proportional to the density of conduction electrons in the bulk of the silicon. In general, the number of conduction electrons is nearly equal to the number of donators (cf. Report 43-12). Therefore, the product of capacity and forward resistance becomes actually larger and the rectification worse, as N_d is decreased. However, an increase in the number of donators is not desirable either. If this number is increased beyond its normal value of about 10^{19} cm^{3} the thickness of the boundary layer will become too small and the contact will then break down very easily in the backward direction (cf. Report 43-12).

A possibility which seems to give a gain would be to try to increase the conductivity without affecting the thickness of the boundary layer. This can be done if the mobility of the electrons in the crystal can be increased. Now the mobility seems to be more or less a constant for each basic substance irrespective of the impurities but it is about 3-5 times greater for Ge than for Si.* If it is possible to overcome the somewhat worse mechanical characteristics of Ge it may be worth while to use this substance as a rectifier in view of the higher mobility of its electrons. However, it will probably be necessary to produce purer Ge before this substance can be used.

An improvement would obviously be obtained if we could reduce the number of donators in the boundary layer of the crystal without reducing the number of conduction electrons in the bulk material. This would require a removal of donators from the surface layer to the interior of the crystal. It seems very difficult to do this by chemical means although this possibility should not be entirely disregarded. An indication of an interesting possibility may be found in the results of Lawson** of the University of Pennsylvania who is at present studying the burn-out of crystals. He has found that the crystals apparently become better rectifiers upon application of a forward voltage equal to about $\frac{2}{3}$ of the voltage required for burn-out. A tentative explanation of this phenomenon is as follows. The voltage applied will raise the temperature high enough to cause diffusion of atoms in the boundary layer. At the same time, the direction of the applied voltage is such that it will move electrons from the semiconductor into the metal (we speak, for simplicity, of an N-type crystal). It will therefore accelerate charge donators into the interior of the semiconductor and will therefore make the surface layer poorer in donators. This will cause an increase of the thickness of the boundary layer and therefore is likely to improve the rectifying characteristic both for D.C. and R.F. This point will have to be studied in more detail; if it is correct, then an equally large backward voltage must make the rectifier worse.

An actual theory of the R.F. rectification as a function of the local oscillator voltage will be developed in the near future. Also a theory of the noise

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*Reports from Purdue and Pennsylvania Universities.

**Private Communication

will be attempted. The ideas presented in this paper are necessarily tentative and are published at this time only in order to accelerate experimental work on this problem. If it were not for this fact these notes would not have been published until experiments on related problems had been completed.

My thanks are due to many members of the Crystal Rectifier Groups of the Radiation Laboratory (Groups 53 and 61) for much valuable information on experimental results. I am also much obliged to Drs. Seitz, Schiff, and Lawson of the University of Pennsylvania, and to Dr. Becker of the Bell Telephone Laboratories for valuable information and discussions.

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10/24/42

Bulk Resistance of a Crystal with a Long Thin Contact

by

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We approximate the contact surface by an ellipse of semidiameters a, b ($a \gg b$) and choose rectangular axes so that the x-y plane contains the contact surface; the long dimension of the contact being along the x-axis. We now define ellipsoidal coordinates ξ, ζ, η by

$$f(\theta) = \frac{x^2}{a^2+\theta} + \frac{y^2}{b^2+\theta} + \frac{r^2}{\theta} = 1 \qquad (11)$$

$$\theta = (\xi, \zeta, \eta)$$

and

$$\xi \geq 0 \geq \eta \; -b^2 \geq \zeta \geq -a^2 .$$

The confocal ellipsoids ξ = constant are the equipotential surfaces of the problem, since for $\xi = 0$, $f(\xi)$ degenerates into the elliptical contact. LaPlace's equation for the potential V becomes:

$$\frac{\partial}{\partial \xi} \sqrt{(\xi + a^2)(\xi + b^2)\xi}\, \frac{\partial V}{\partial \xi} = 0 .$$

Thus

$$V = A \int_0^{\xi} \frac{d\xi}{\sqrt{(\xi + a^2)(\xi + b^2)\xi}} \qquad (12)$$

where A is an integration constant.

This makes $V = 0$ at the whisker contact and if $V = V_0$ (the applied potential) at $\xi = \infty$

$$A^{-1} = \frac{1}{V_0} \int_0^{\infty} \frac{d\xi}{\sqrt{(\xi + a^2)(\xi + b^2)\xi}}$$

Thus (12) becomes, using Pierce's tables (No. 543),

$$V = \frac{V_o}{K} \, sn^{-1} \left(\sqrt{\frac{\xi}{\xi + b^2}} \; , \; k \right) \qquad (13)$$

when

$$k = \sqrt{1 - b^2/a^2} \qquad (13a)$$

and

$$K = sn^{-1} (1, k) \qquad (13b)$$

is the complete elliptic integral of the first kind with modulus k. Now if r is the radial distance from the center of the contact: as $r \rightarrow \infty$, $\xi \rightarrow r^2$ and

$$V \rightarrow V_o \left(1 - \frac{a}{K r} \right) .$$

The current is thus

$$I = 2\pi \, r^2 \sigma \frac{\partial r}{\partial r}$$

$$= \frac{2\pi \sigma a}{K} \; V_o \; , \text{ when } \sigma = \text{conductivity},$$

and the resistance is

$$R = \frac{K}{2\pi \sigma a} \qquad (14)$$

This result is general.

So far, we have not used the condition that $a >> b$. In fact, $a = b$, $K = \frac{\pi}{2}$ and $R = \frac{1}{4 \sigma a}$, the well-known result for a circular contact of radius a. If $a >> b$, $k \sim 1$, and $K \rightarrow \ln 4\frac{a}{b}$.

So that for a long thin contact

$$R = \frac{\ln 4 \; a/b}{2\pi \sigma \; a} \qquad (15)$$