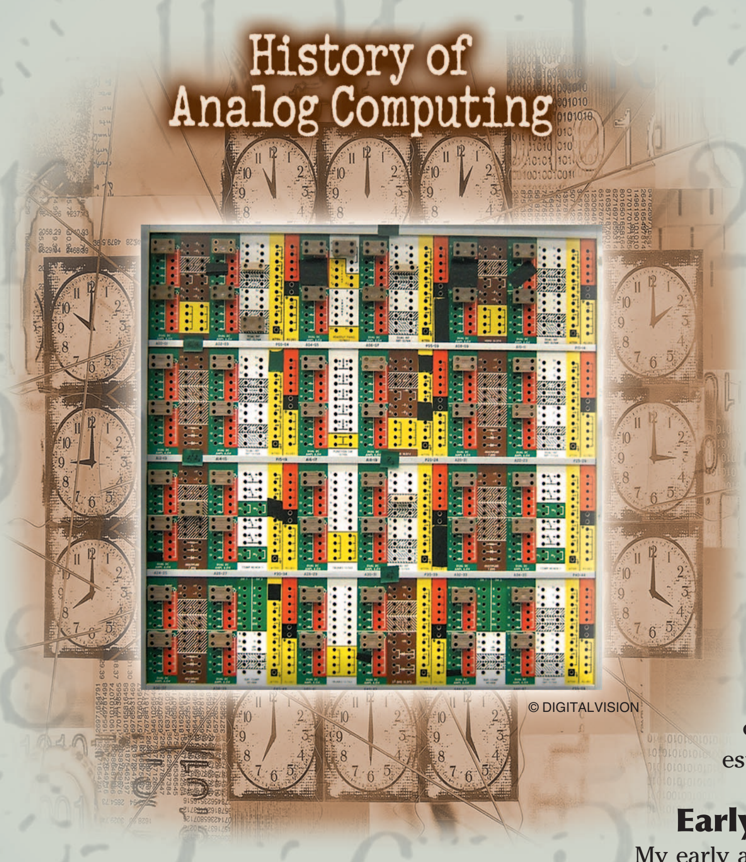
A Reconstruction of the Differential Analyzer in Meccano

The challenges of torque amplifiers, integrators, and backlash

By Tim Robinson



I was first introduced to Meccano, a child's educational construction set created in the United Kingdom, at about the age of six and quickly became fascinated with it as a medium for constructing working mechanisms and small machines. Over the next ten years or so, I gathered quite a large collection. I first attempted to construct a differential analyzer in Meccano around 1971. I had just encountered calculus in high school and at the same time I started to develop an interest in computers. One of the first books I read on computers included a chapter on analog computation, and on the differential analyzer in particular. Significantly, the book briefly mentioned that simple differential analyzers had been constructed in Meccano in the 1930s. So began an interest that has remained with me for more than 30 years.

Early Attempts

My early attempts were not very successful because of the difficulty of constructing functioning torque amplifiers. When Hartree and Porter built the first Meccano differential analyzer at Manchester University, their goal was to build a working machine quickly. They used Meccano simply because it was readily available and allowed them to avoid designing and custom machining most of the required parts. However, Hartree and Porter felt no constraint to stay within the limits of the Meccano system. In particular, they did not believe that adequate torque amplifiers could be made without custom machining. In contrast, as a Meccano enthusiast approaching the same subject, finding a solution within the system to what at first seems an insurmountable problem was part of the challenge.

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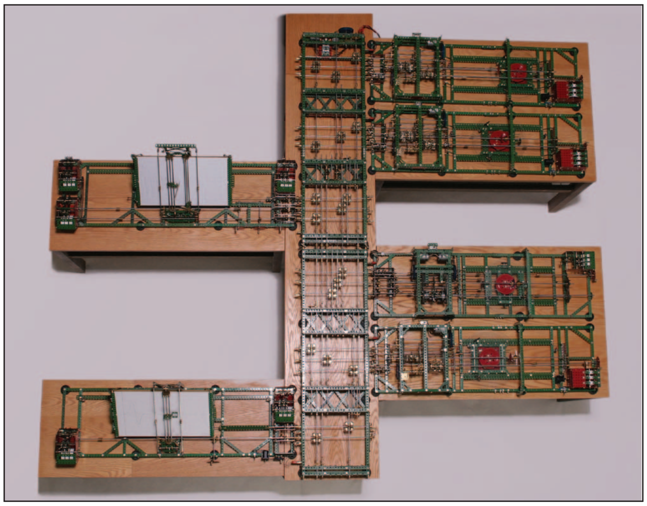


Figure 1. *Aerial view of the machine. Down the center is the interconnect section in which shafts and gearing are placed to "program" the machine for a particular equation. On the left are the input table and dual output table, and on the right are four integrators. Each of the functional units can be detached from the central interconnect for maintenance and transportation. (Photo: Michael Baxter.)*

Having failed in that first attempt, I revisited the differential analyzer periodically. The main obstacle to making the amplifier is the lack of any form of concentric shafting system. In Bush's prototype machine, the amplifiers use drums that rotate on hollow shafts rigidly fixed to the framework. This arrangement allows the input and output shafts to pass through with no friction coupling directly from the drums to the input. Copying the general layout in Meccano, but without the fixed sleeve between the input shaft and the drum, inevitably means that an unacceptable torque is coupled from one of the drums directly to the input shaft. This coupling causes the integrator wheel to steadily slip in the direction of the drum's rotation. By adding a counter-rotating element to the input shaft to at least partly compensate for the undesirable friction, I succeeded in making a simple two-integrator demonstration model. Although the amplifiers had only a single stage, with the use of ground glass instead of plain glass discs to increase the friction available at the integrator wheel, the mechanism could be made to solve the second-order equation for simple harmonic motion at least qualitatively.

Figure 2. *Torque amplifier stage. The output shaft to the left carries both rotating drums. The input shaft runs parallel on the right, carried in rotating bearings, and couples to the input arms through a pair of gears.*

A Successful Torque Amplifier

The machine I have today (Figure 1) contains about 16,000 individual parts, about half of which are nuts, bolts, and washers. Construction started about four years ago, after a flash of insight led me to a robust solution to the amplifier problem. Instead of arranging the input and output shafts of the amplifier to be colinear with one drum on each as Bush had done, I arranged the input shaft to run parallel to the output, with both drums running on the output shaft (Figure 2). The input is coupled to the input arms of the amplifier by means of a pair of gears, one fixed on the input shaft and the other free to rotate on the output shaft between the drums. Since the output shaft has the benefit of the amplification of torque compared to the input, any residual friction from the drums running directly on the output shaft is unimportant. While this feature was the key to constructing a working amplifier within the constraints of the Meccano system, another critical element turned out to be the choice of material for the friction bands. Particularly in the second stage, the bands are under heavy load and can heat up significantly from the friction. If the band material is not stable, the critical adjustment of tension in the bands will be upset during operation. If the material stretches, excessive backlash is introduced and the bands are likely to slip off the drums. If the material shrinks, then the bands tighten, increasing friction further in a cycle of positive feedback that leads to the amplifier seizing up. After experimenting with many materials, I finally selected Dacron cord, a material favored by kite flyers because of its light weight, great strength, and resistance to stretching. Dacron performs extremely well for my torque amplifier design.

With these fundamental issues addressed, I evaluated a number of design variations, varying in minor details mostly to improve strength and rigidity. I steadily improved the performance and finally arrived at a two-stage design that runs reliably for days at a time (Figure 3). The rest of the machine is relatively straightforward by comparison.

Integrators

Integrators are the principal functional units of a differential analyzer, and a few details of the integrators in the model are worth highlighting (Figure 4). The output of an integrator is taken from the integrating wheel, which rests on the disk under very light pressure. The available torque is further reduced by any vibration when the machine is in operation. To ensure there is no slipping, friction on the shaft that couples the integrating wheel to the input of the torque amplifier must be minimal. Friction reduction is accomplished by mounting the shaft in rotating bearings, driven from the output of the torque amplifier. Because the bearings rotate at the same rate as the shaft, friction is essentially reduced to zero. If the integrator wheel is lifted clear of the integrator disk and set in motion, the wheel will spin almost indefinitely. Although the rotating bearings introduce the possibility of feedback-path coupling from the output of the amplifier back to the input, there appears to be no evidence of instability in practice.

The integrator disks are made from window glass cut to a circle with a diameter of 11 in and attached to a standard Meccano component using hot glue. Glass is a convenient, readily available material that provides a hard-wearing, flat surface. A large diameter provides greater accuracy, and this size was chosen because the longest Meccano screwed rod, used for the carriage lead screw, is 11-1/2 in. The disk is fairly heavy and must rotate very freely. To facilitate free rotation, the builders of the early Meccano differential analyzers again reached outside the system and used commercial ball bearing assemblies to carry the disk. I adopted an effective alternative solution, namely, supporting the weight of the disk on three small wheels positioned near the periphery. The driving shaft provides centering and the driving torque, but does not need to support the weight. The disk can be lifted off for cleaning.

The integrator carriage must be accurately positioned to represent the value of the function being integrated. Any play in the lead screw directly affects the accuracy of the machine. To avoid play, two nuts are used on the screw, forced apart by a compression spring. The force of the spring ensures that the defining nut always contacts the same side of the lead-screw threads. This device was used by Bush in the prototype and he referred to it as a "lashlock." To eliminate end float in the bearing, a second spring preloads the bearing of the shaft that drives the screw.

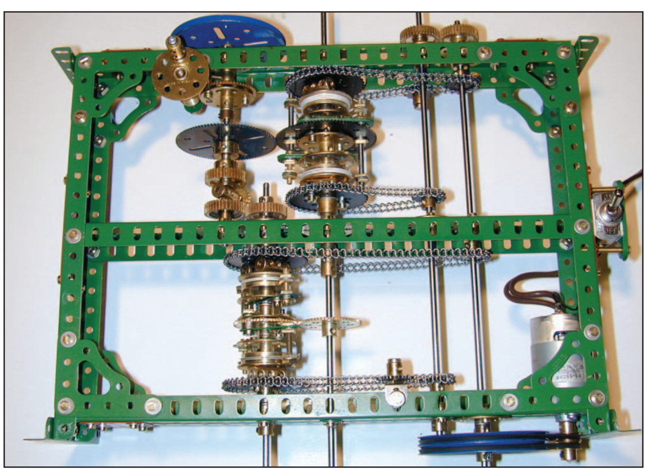


Figure 3. *Two-stage amplifier. This amplifier has two similar stages connected in series for an overall gain of approximately 10,000. A "frontlash" unit in the form of a differential gear can be seen at the upper left, in the path between the two stages. The amount of backlash to be compensated is set by using the small hand crank.*

Backlash

In all mechanical systems, there is unavoidable backlash. This fact is particularly true in the case of Meccano, which, being essentially a toy, is not manufactured to especially close tolerances. Starting from an integrating wheel, there is backlash in the torque amplifier as well as in the gears in the shafts connecting the output of the amplifier to either a lead screw or the disk of another integrator. In an analog machine, where variables are represented by shaft angles, backlash represents a significant source of error. In his prototype, Bush introduced something he called a "frontlash" unit to compensate the backlash to first order. These units took the form of epicyclic gear assemblies, which could be inserted as required in the interconnect. Whenever the direction of motion of the input to the frontlash unit reverses, a small amount of extra motion is added to the output to compensate for the motion lost to the backlash. The frontlash unit proved to be another difficult item to model in Meccano, not because of any problem in principle but because of the difficulty of creating a unit that can operate smoothly under the heavy loading of actual operation. After a number of unsatisfactory attempts, I solved this problem by using a differential-based design, located between the two stages of each torque amplifier. By positioning the differential there, the frontlash unit operates under only a light loading since it is followed by the gain of the second amplifier stage. The amount of compensation is adjustable and can be set for a particular setup by measuring the backlash empirically.

Input/Output Tables

In Bush's prototype, the input and output tables each had a fixed bed over which the cross hair or recording pen could be driven in two dimensions by a pair of lead screws. In Meccano, it is much easier to arrange for the cross hair or pen to move along a fixed axis and have the bed of the table move bodily in the perpendicular direction beneath it. This arrangement was used by Hartree and

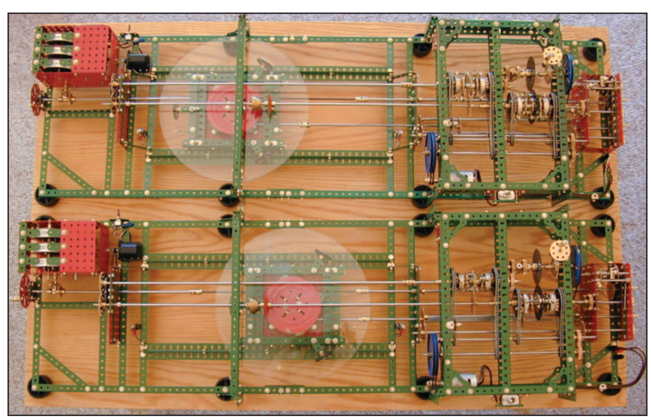


Figure 4. *A pair of integrators, each incorporating a two-stage torque amplifier. Digital counters on the carriage lead screws facilitate accurate setting of the initial conditions.*

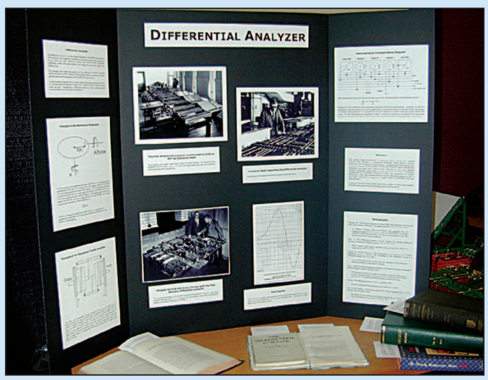
Meccano Differential Analyzer at VCF 7.0

On 6–7 November 2004, the Vintage Computer Festival held its seventh annual conference and exhibition, hosted by the Computer History Museum in Mountain View, California. This year's event attracted some 15 speakers, 40 exhibitors, and 500 attendees. The festival brings together enthusiasts, historians, and interested members of the general public. The event has a broad focus, covering every aspect of computer hardware and software, providing the subject is at least 10 years old.

Multiple conference tracks covered topics as diverse as "The IBM 360 Evolution and Revolution" and "Using Vintage Computers in Computer Forensics." Exhibits ranged from a display of 60 years of palm-sized computational devices, through a broad assortment of early and long-forgotten personal computers and workstations, to a recreation of a 1972 DEC-based data center, complete with multiple mag-tape drives, line-printers, and an array of interactive terminals, running an equally ancient version of UNIX.

Most of the exhibits were entered for judging in one of six classes. To be eligible for judging, exhibits must be fully operational, must be running period software as appropriate, and must operate without error at the appointed time before the judges. The author entered his Meccano model of Bush's 1930 differential analyzer in the class "Re-creation, Emulation, or Contemporary Enhancement." The displayed machine had four integrators, input table, and dual-output table and, for demonstration purposes, was configured to solve a forced, damped simple harmonic oscillator equation. The Meccano model operated continuously and reliably for both days of the event. Although the chosen setup used only three of the available four integrators, the setup was simple enough that visitors with an engineering or scientific background could quickly relate to it. An accompanying display board covered the history of the prototype, reviewed the principles of operation, and documented the demonstration setup. Original papers and articles demonstrating comprehensive research completed the exhibit.

Two young boys are enthralled. Although too young to understand what the machine does, the children are captivated by the vast number of moving parts.



Supporting display board. The display includes pictures of Bush's prototype, Hartree and Porter's first Meccano machine, principles of operation, a setup diagram for the equation being run for demonstration, and a brief bibliography.

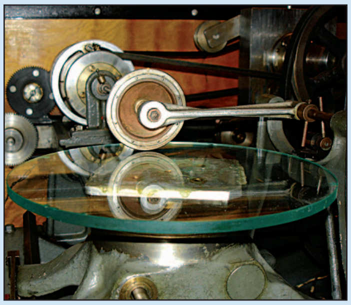
The author at the input table while visitors look on. The exhibit drew numerous onlookers who were fascinated by the operation of the machine.

A panel of judges assessed each exhibit according to a set of formal criteria. Awards were given for first, second, and third place in each class. In addition, a number of special awards (independent of class) were offered. The Best in Show award was taken by an outstanding exhibit of products from The Digital Group covering the period 1974–1978. The differential analyzer received 1st place in its class and claimed special awards for "Best Technology: Analog" and "Best Technology: Nonelectronic."

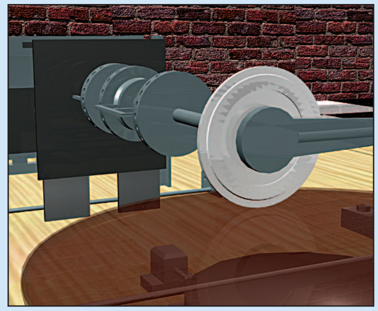
The differential analyzer exhibit proved popular. Experts and the general public, especially children, were equally fascinated. The combination of visible mechanical operation and the medium of its construction immediately drew people toward the machine. Although the machine is inherently slow, most people were willing to take the time to watch the solution steadily unfold, and many requested detailed explanations of aspects of its operation. The mechanical torque amplifiers received particularly close attention.

At the conclusion of the show, an additional prize, the "People's Choice Award" was decided by popular vote of the exhibition visitors. The differential analyzer stole this one handsomely!

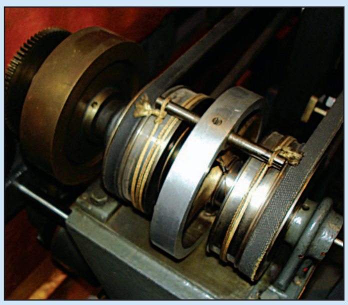
Virtual Differential Analyzer Reconstruction



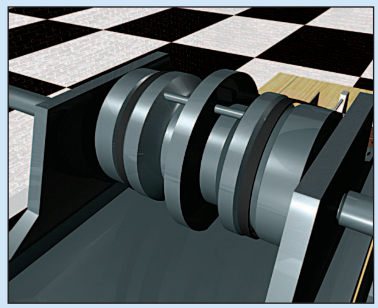
Original wheel-and-disc integrator from Bush's differential analyzer on display at the MIT Museum. This view shows the glass disc and the metal wheel used to implement integration in the computer.



Animated wheel-and-disc integrator. The animated integrator demonstrates the operation of this clever mechanism, providing insight into the operation of the differential analyzer.



Original Nieman torque amplifier on display at the MIT Museum. This torque amplifier, based on contra-rotating capstans, amplifies the output from the integrator wheel and drives the output gears of the integrator box.



Animated torque amplifier. By watching a short animation of the torque amplifier in operation, students can observe and learn the principles behind the amplifier's operation.

Mechanical differential analyzers are often praised for their educational value. Vannevar Bush tells the story of a draftsman who learned differential equations in mechanical terms from working on the construction and maintenance of the MIT differential analyzer (see [1]). After the last differential analyzer at MIT was decommissioned, Warren Weaver wrote to Samuel Caldwell:

> [I]t seems rather a pity not to have around such a place as MIT a really impressive Analogue computer; for there is vividness and directness of meaning of the electrical and mechanical processes involved... which can hardly fail, I would think, to have a very considerable educational value. A Digital Electronic computer is bound to be a somewhat abstract affair, in which the actual computational processes are fairly deeply submerged. [2]

In the place of an expensive reconstruction of the original high-precision machinery [3], we have completed a virtual reconstruction of the Bush differential analyzer. Based on original descriptions and photographs of the computer [4], a graphical model of the computer has been constructed using Maya Complete 6.0, a three-dimensional (3-D) modeling software package.

The analyzer is modeled with a physical description of spline curves and meshes of polygons. The scene is illuminated with three sources of light, one directional source and two ambient sources, to enhance the appearance of the components. Texture maps of metal, wood, brick, and floor tiles are used to add detail to the rendered objects.

The complete model will be animated to show the operation of the machine solving differential equations. Several days of computer time are needed to produce the individual frames for a single animation. The final video is assembled using Adobe AfterEffects and compressed using the DivX codec. Preliminary images and videos from this project are available on the Web [5].

References

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[5] K.H. Lundberg and V. Pereverzev, "Vannevar Bush's differential analyzer" [Online]. Available: http://web.mit.edu/klund/www/analyzer/

—Kent Lundberg and Vitaliy Pereverzev

Porter in the first Meccano model, and adopted in my design. I have, however, tilted the table beds to 45° to make the operator's task more pleasant. The tables are sized to accept standard 11 in × 17 in graph paper.

Overall Layout

On this incarnation of the model, I set out with the goal of reproducing Bush's machine configuration as closely as possible. In particular, the design anticipates a total of six integrators, although so far I have completed only four. The prototype used a monolithic construction literally bolted to a concrete floor. Similarly, when Hartree and Porter built the first Meccano model, they were not concerned with portability, and used a large sheet of plywood as a base to which the various Meccano parts could be firmly screwed down. I decided that I wanted a machine that could easily be transported and reconfigured for demonstration purposes (see "Meccano Differential Analyzer at VCF 7.0."), so I constructed some lightweight custom tables and arranged the framework of the machine to be modular, resting on the tables on vibration-absorbing rubber feet. Although this approach requires far more Meccano parts, modularity is much more in the true spirit of the medium. It also makes maintenance easier, since individual units can be readily removed to a workbench.

I wanted to create more than a simple demonstration model. Specifically, my goal was to build a machine to simplify the work required to set up a new equation and perform repeated runs with varying initial conditions; that is, a machine that has good "usability." In this respect, the 1930s Meccano models fell somewhat short of the full-scale prototype, sometimes requiring modifications on the fly as new equations were being set up. Following Bush's layout, my machine has a central interconnect section (which can be split into subsections for transport), into which the integrators and input/output tables can be plugged. One improvement that subsequent full-scale machines adopted relative to Bush's prototype was the provision of a second level of bus shafts, which adds considerable flexibility in setting up a particular equation. I decided to adopt a similar scheme.

Scaling up the machine from a simple two-integrator demonstration model introduces new challenges. The much greater amount of interconnect required for a realistic configuration increases the amount of friction and, with it, the loading on the torque amplifiers. Some bus shafts often need to run the entire length of the machine, which is over 10 ft for the six-integrator configuration. This scale calls for great care and patience in construction to achieve sufficient rigidity and accurate alignment of components. The bus shafts are in fact made up of much shorter axles with couplings between the sections. This design is essential to allow "programming" to be localized. Each segment can be removed independently to permit easy changing of gears and interconnection points, just as on Bush's prototype.

Once an equation has been set up on the machine, initial conditions must be set prior to each run. To facilitate initialization, all lead screws can be disconnected from the main interconnect by friction clutches, thus permitting them to be set to the appropriate starting positions. Each lead screw is coupled to a decimal rotation counter, which provides settings that are accurate to a fraction of a turn. A small motor drive is provided at each integrator lead screw to reduce the amount of hand cranking needed to set initial conditions. All lead screws include limit switches, which cut power to the independent variable motor in the event that any unit exceeds its mechanical limits. Without limit switches, excessive rotation can induce a severe strain on the torque amplifier driving the lead screw that hits the limit, usually resulting in damage.

Future Plans

I plan a number of enhancements to my design. Foremost is the addition of more integrators. Realistic problems require more integrators than the simple mathematical order of the equation because additional integrators are used as function generators and sometimes as multipliers. Indeed, the central table was initially sized assuming an eventual configuration with six integrators. Next in line is a second input table configured to allow optional use as a multiplier. With these additions, the capacity of the machine will be quite comparable to Bush's prototype and to other reconstructions (see "Virtual Differential Analyzer Reconstruction").

Tim Robinson (tbr00@pacbell.net) retired in 2003 from Broadcom Corporation, where, as senior director of engineering, he was responsible for the development of Broadcom's range of WiFi (802.11a/b/g) wireless networking chipsets. He holds bachelor's and master's degrees in physics from Oxford University. He entered the computing field in 1980 in the United Kingdom, where, as cofounder of High Level Hardware Ltd., he designed a user-microprogrammable computer system for developing novel programming languages. In 1989, he moved to the San Francisco Bay Area, where he has held senior engineering positions at a number of Silicon Valley startup companies. He maintains a strong interest in the early history of computing, particularly mechanical computing devices, and is actively involved in the restoration of these early machines and in the construction of working replicas of Charles Babbage's conceptual designs. Additional interests include music, Meccano, and current developments in physics and cosmology. Tim can be contacted at 216 Blackstone Dr., Boulder Creek, CA 95006 USA.